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Tangible Mathematics for Grade 7 Learners: Design and Validation of Manipulatives for Rational Number Concepts

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Abstract

This study presents the development and evaluation of tangible mathematics, investigating how these materials enhance grade 7 learners' engagement, understanding, and problem-solving skills in number and algebra. Following a needs assessment that identified rational numbers as the most challenging topic, this phase focused on designing and validating physical manipulatives grounded in established learning theories. The development followed a design-based research approach with iterative refinement cycles. Tangible Mathematics —including fraction circles, bars, tiles, and operational tiles manufactured from clear acrylic—were developed with systematic color-coding, clear fraction markings, and movable connections. Validation involved five mathematics education experts using Department of Education assessment rubrics, yielding exceptional scores across content quality (37.8/40), technical accuracy (16/16), and instructional design (22/24). Teacher feedback highlighted the materials' effectiveness in supporting concept visualization and improving attitudes toward mathematics, while recommending physical enhancements. Preliminary testing with 84 grade 7 learners showed significant performance improvements in the experimental group using the manipulatives compared to the control group. Both quantitative assessment data and qualitative classroom observations confirmed that the tangible materials successfully enhanced learner understanding, engagement, and problem-solving abilities in rational number operations. These findings provide a strong foundation for subsequent implementation of the tangible mathematics approach in grade 7 classrooms.

Keywords: manipulatives, design-based, understanding, problem-solving, engagement

1. Introduction

The transition to grade 7 mathematics represents a critical juncture in students' mathematical development, characterized by significant shifts from concrete to abstract thinking. During this period, learners face increasing cognitive demands as they navigate the bridge between arithmetic and algebraic reasoning (Wilkie & Sullivan, 2017). Despite ongoing curricular reforms, persistent challenges in mathematical understanding continue to impact learner achievement, particularly in fundamental areas such as rational numbers and algebraic concepts (OECD, 2023).

This study presents the development phase of a three-phase research project investigating how tangible mathematics can enhance grade 7 learners' engagement, understanding, and problem-solving skills. The research addresses a significant gap in current educational practice – while manipulatives and tangible approaches have demonstrated effectiveness in early mathematics education, their systematic implementation at the grade 7 level remains limited, particularly for abstract concepts like rational numbers. By identifying specific mathematical challenges and resource needs, this study establishes a foundation for developing targeted interventions that bridge concrete and abstract understanding.
The development phase builds directly on findings from the needs assessment phase, which revealed that rational numbers were unanimously identified by teachers as the most challenging topic for Grade 7 learners, with 78.6% of learners rating these operations as "very difficult." The stark contrast between learners' confidence with visual-spatial concepts (mean rating 4.77/5.00) versus rational numbers (1.12/5.00), coupled with all learners (100%) expressing preference for visual aids and hands-on materials, provides clear direction for the development of tangible mathematics interventions.

2. Literature Review

Theoretical Foundations for Tangible Mathematics

The theoretical foundations for tangible mathematics draw from several fundamental learning theories that support hands-on approaches to mathematical instruction. Constructivism, as developed by Piaget (1952) and elaborated by Bruner (1966), provides a crucial foundation for understanding how learners actively construct mathematical knowledge through physical experiences. This theoretical perspective suggests that concrete manipulation of mathematical concepts can support the development of deeper understanding, particularly during the transition to abstract thinking.

Cognitive Load Theory, introduced by Sweller (1988), offers valuable insights into how tangible approaches might support mathematical learning. The theory suggests that by providing external, physical representations of mathematical concepts, tangible materials can reduce cognitive load, allowing learners to focus more mental resources on understanding fundamental principles. This theoretical framework is particularly relevant when considering how to design materials that support rather than overwhelm learners' cognitive processing capabilities.

The theory of Embodied Cognition, as presented by Lakoff and Núñez (2000), provides additional support for tangible approaches by highlighting the fundamental connection between physical experiences and abstract understanding. Their research suggests that even highly abstract mathematical concepts are grounded in bodily experiences, supporting the use of concrete materials in developing mathematical understanding.

Bridging Number and Algebraic Understanding

The theoretical connections between numerical and algebraic understanding provide important foundations for grade 7 mathematics instruction. Kaput et al. (2017) developed a comprehensive theoretical framework that reconceptualizes algebra not as a separate domain but as a strand of mathematical thinking that should develop alongside arithmetic understanding from early education onward. Their theoretical model identifies three primary strands of algebraic reasoning: the study of structures and systems abstracted from computation, the study of functions and relations, and the application of modeling languages for expressing and supporting reasoning about situations.

Carraher et al. (2006) further developed this theoretical integration through their research on how children develop algebraic thinking through generalized arithmetic. Their theoretical approach emphasizes that "arithmetic has an inherently algebraic character when it is conceived in terms of operations on sets rather than simply as computation" (p. 92). This perspective suggests that the arithmetic-to-algebra gap often observed in Grade 7 students results not from developmental constraints but from instructional approaches that emphasize computational procedures over structural understanding.

Design Principles for Mathematical Manipulatives

The design of mathematical manipulatives requires careful consideration of several key principles identified through research. Moyer-Packenham and Westenskow (2013) conducted a comprehensive analysis of manipulative design features, finding that effective materials must align closely with mathematical concepts while remaining accessible to learners. Their research emphasizes the importance of clear connections between physical representations and mathematical ideas.

The theoretical foundations for effective manipulative design have been explored by several researchers. Laski et al. (2015) examined what makes mathematics manipulatives effective by drawing lessons from cognitive science and Montessori education. Their analysis identified four key principles for manipulative effectiveness: (1) design that connects directly to mathematical concepts, (2) explicit scaffolding that bridges concrete and abstract representations, (3) consistent use over time, and (4) gradual removal of the physical support.

Comprehensive conceptualizations of manipulatives in mathematics education have been provided by Bartolini and Martignone (2020). Their research establishes a theoretical framework for understanding how manipulatives function in the learning process, distinguishing between their roles as tools for representation, exploration, and concept development. This nuanced understanding of manipulative functions provides crucial guidance for designing materials that support specific learning objectives.

3. Methodology

Research Design

The development of tangible mathematics materials for grade 7 algebra instruction was guided by design-based research (DBR), a methodological approach that systematically addresses the complex challenge of enhancing mathematical learning through innovative instructional strategies. This approach was selected for its ability to bridge theoretical research and practical classroom application while allowing for continuous improvement based on feedback and evaluation.

The design-based research process involved four main iterative phases in developing the manipulatives:

Initial Design Phase: Conceptualization based on needs assessment findings and development of preliminary designs using Lightburn software, with a specific focus on addressing the identified learning challenges in fractions.

- Prototype Development Phase: Creation of initial prototypes using chipboard material, allowing for testing of dimensions and proportions, and refinement of design elements.
- Panel Evaluation and Refinement Phase: Assessment by a panel of five mathematics educators using the Department of Education's structured assessment rubric for manipulatives, collection and analysis of panel feedback, and implementation of modifications based on their recommendations.
- Final Production Phase: Implementation of the refined designs using acrylic materials, application of color coding and labeling systems, and production of the complete set of manipulatives.

Participants and Subject of the Study

The development phase brings together multiple stakeholder groups to support the development and validation of tangible mathematics materials:

- Panel Evaluators: Five mathematics educators providing critical assessment of material design and educational effectiveness
- Teacher Reviewers: Experienced grade 7 mathematics teachers offering practical insights into implementation feasibility
- Learner Testers: 84 grade 7 learners providing essential feedback on material usability and engagement

For the preliminary testing, the 84 grade 7 learners were divided into control (n=37) and experimental (n=47) groups based on their intact class sections. The distribution between the groups showed the experimental group containing 56% of total participants and the control group having 44%.

Research Instruments

- Panel Evaluation Forms: Incorporated detailed rubrics for content evaluation, technical accuracy assessment, and instructional design review, ensuring comprehensive assessment of material quality.
- Teacher Feedback Forms: Focused on material usability assessment, implementation feasibility review, and suggestion documentation, providing practical insights for refinement.
- Learner Testing Protocols: Included structured observations of material interaction, understanding check assessments, and engagement evaluation forms, offering direct evidence of material effectiveness from the learner perspective.

Tangible Mathematics: The study developed and utilized specific manipulatives designed to support rational number understanding, including:

- Fraction Circles: Clear acrylic manipulatives with 5-inch diameter, designed for exploring fraction relationships and equivalence
- Fraction Bars: 18.25-inch by 1.5-inch clear acrylic bars for fraction comparisons and operations
- Fraction Tiles: Clear acrylic tiles for rational number operations
- Operational Tiles: 2-inch by 1.5-inch tiles featuring mathematical symbols and operations

Figure 1: Tangble Mathematics

Ethical Considerations

This study maintains strict ethical standards throughout all phases to protect participants and ensure responsible research practices.

Development Ethics

The study maintains ethical standards throughout the development process. The development process adheres to ethical material development practices, ensuring content appropriateness and cultural sensitivity. Learner testing environments are carefully monitored to ensure participant safety and comfort. All feedback and testing data are handled confidentially, with careful attention to protecting participant privacy while maintaining the integrity of the development process.

Data Collection and Analysis

Data collection focused on gathering detailed feedback for material development and refinement through panel validation, teacher review, and learner testing. The analysis employed both quantitative and qualitative methods:

- Panel Evaluation Analysis: Examination of content validation scores, synthesis of panel recommendations, and documentation of refinement needs.
- Teacher Feedback Analysis: Examination of usability ratings and thematic analysis of suggestions.
- Learner Testing Analysis: Analysis of interaction patterns, assessment of material effectiveness through observational data and direct feedback, and statistical comparison of control and experimental group performance.

4. Results and Discussion

Development of Tangible Mathematics

The developed manipulatives, "Tangible Mathematics," consist of two main component types: fraction-based manipulatives and operational tiles, each designed with specific features to support mathematical learning. All components were manufactured using clear acrylic material, chosen for its durability and visual clarity.

The design incorporated several key features to enhance educational effectiveness:

- A systematic color-coding system across all fraction pieces
- Movable connections enabling learners to align and manipulate pieces during problem-solving
- Clear fraction markings applied using acrylic spray paint
- Precise laser-cutting techniques ensuring accurate mathematical representations

Evaluation of the Manipulatives: Tangible Mathematics

The evaluation of the tangible mathematics demonstrated consistently high performance across all indicators. The content evaluation (Factor A) yielded an impressive score of 37.8 out of 40 points, with all indicators achieving ratings within the highest "Very Satisfactory" category (3.26-4.00), as shown in Table 1.

Table 1: Evaluation in Terms of Factor A (Content)

Factor A. Content	Mean Rating	Descriptive Interpretation
1. Content reinforces, enriches, and/or leads to the mastery of certain learning competencies for the level and subject it was intended.	3.8	Very Satisfactory
2. Material has the potential to arouse interest of the target users.	4	Very Satisfactory
3. Facts are accurate.	4	Very Satisfactory
4. Information provided is up – to – date.	4	Very Satisfactory
5. Visuals are relevant to the text.	3.6	Very Satisfactory
6. Visuals are suitable to the age level and interests of the target user.	3.6	Very Satisfactory
7. Visuals are clear and adequately convey the message of the subject or topic.	3.8	Very Satisfactory
8. Typographic layout/design facilitates understanding of concepts presented.	3.8	Very Satisfactory
9. Size of the material is appropriate for use in school.	3.6	Very Satisfactory
10. Material is easy to use and durable.	3.6	Very Satisfactory
Total Points	37.8	Passed

Note: Score must be at least 30 points out of a maximum 40 points to pass this criterion.
Very Satisfactory, 3.26 – 4.00; Satisfactory, 2.56 – 3.25; Poor, 1.76 – 2.50; and Not Satisfactory, 1.00 – 1.75.

For Factor B, which focused on identifying potential errors, the materials received a perfect score of 16 out of 16 points, signifying the complete absence of conceptual, factual, grammatical/typographical, and other technical errors, as shown in Table 2.

Table 2: Evaluation in Terms of Factor B (Other Findings)

Indicators	Mean Rating	Descriptive Interpretation
1. Conceptual errors	4.00	Not Present
2. Factual errors	4.00	Not Present
3. Grammatical and/or typographical errors	4.00	Not Present
4. Other errors (i.e., computational errors, obsolete information, errors in the visuals, etc.)	4.00	Not Present
Total	16	Passed

Note: Score must be at least 16 points out of a maximum 16 points to pass this criterion.
Not Present, 3.26 – 4.00; Present but with very minor and must be fixed, 2.56 – 3.25; Present and requires major redevelopment, 1.76 – 2.50; and Poor, 1.00 – 1.75.

The evaluation of Factor C, examining both instructional and technical design aspects, achieved a total score of 22 out of 24 points, which substantially exceeds the minimum passing requirement of 18 points. The results demonstrate strong performance across both design categories, with most indicators falling within the "Very Satisfactory" range, as shown in Table 3.

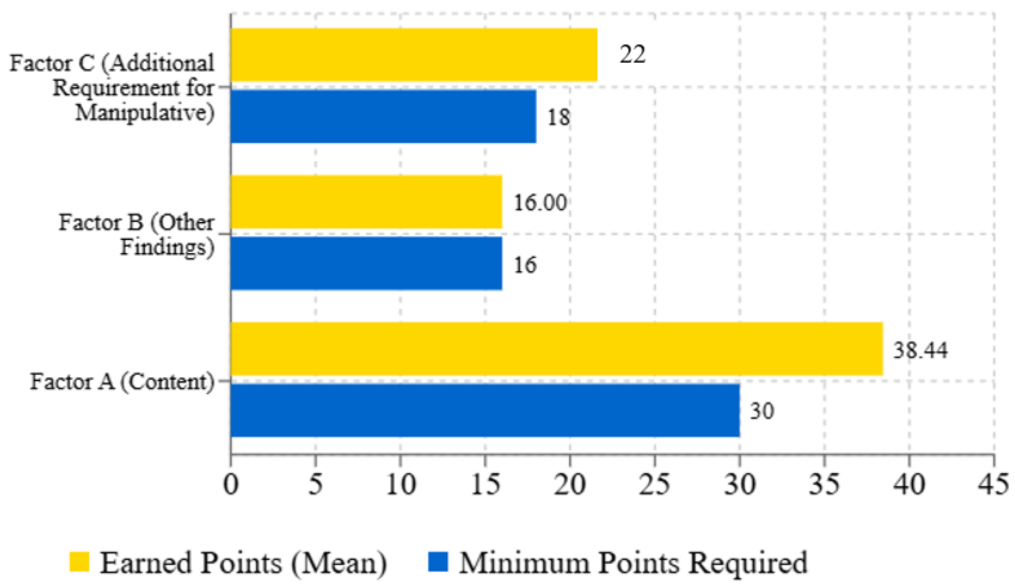
Table 3: Evaluation in Terms of Factor C (Additional Requirements for Manipulative)

Indicators	Mean Rating	Descriptive Interpretation
Instructional Design		
1. Adequate support material is provided.	3.4	Very Satisfactory
2. Activities are summarized, extension activities are provided.		
3. Suggested activities support innovative pedagogy.	4.0	Very Satisfactory
Technical Design		
4. Manipulative is safe to use.	3.6	Very Satisfactory
5. Size and composition of manipulative is appropriate for intended		Very Satisfactory
audience.	4.0	Satisfactory
6. Suggested manual tasks within the activities are compatible with the motor skills of the intended users.	3.2	Very Satisfactory
	3.8	
Total	22	Passed

Note. Score must be at least 18 points out of a maximum 24 points to pass this criterion.
Very Satisfactory, 3.26 – 4.00; Satisfactory, 2.56 – 3.25; Poor, 1.76 – 2.50; and Not Satisfactory, 1.00 – 1.75.

Overall, the evaluation results shown in Figure 2 indicate that the tangible mathematics manipulatives not only met but substantially exceeded the minimum requirements prescribed by the Department of Education for manipulatives across all evaluation areas.

Figure 2: Summary of Ratings for the Areas of Evaluation



Teacher Feedback Analysis

The analysis of teacher feedback shown in Table 4 revealed five major themes: Physical Design Features, Instructional Applications, Learning Impact, Extended Applications, and Areas for Enhancement.

Teachers identified several strong features of the manipulatives, including precise sizing, attractive color scheme, and tangible/detachable nature. They emphasized how the manipulatives helped learners visualize fractions and facilitated learner engagement through their interactive design.

For instructional applications, teachers identified diverse ways to integrate the manipulatives into their teaching practice, particularly for preparatory activities, motivation, and group work. They noted significant positive effects on learner learning, especially in helping learners better understand mathematical concepts and reducing negative attitudes toward fractions.

Teachers also suggested potential improvements, primarily focused on physical aspects such as durability and size, with recommendations for larger dimensions to enhance usability in classroom settings.

Table 4: Thematic Analysis of Teachers' Feedback with Supporting Quotes

Theme Category	Key Elements	Supporting Quotes
Physical Design Features	• Precise sizing • Attractive colors • Tangible/detachable components • Easy to manipulate	• "The size is very precise" • "The color is attractive to students" • "It can easily be manipulated and colorful" • "Color combinations and the tangible/detachable manipulatives"
Instructional Applications	• Preparatory/motivational activities • Group learning activities • Fraction operations teaching • Concept visualization support • Recreational learning tool	• "It can be used as preparatory activity or motivation before the lesson proper" • "Can be used best in group activities" • "the manipulatives can be a recreational tool as motivation in the lesson" • "help the students visualize fractions"
Learning Impact	• Enhanced concept understanding • Improved attitude toward fractions • Concrete learning experiences • Learner engagement • Concept visualization	• "This will help students not to hate fractions" • "Concrete learning among learners" • "education value to help students appreciate and understand more in fractions" • "Measurement"
Extended Applications	• Measurement concepts • Algebraic expressions • Different fraction forms • Improper fractions • Dissimilar fractions	• "Can be used in Rational Algebraic Expressions" • "teaching improper or dissimilar fractions" • "Best if students can manipulate other fraction forms"
Areas for Enhancement	• Durability concerns • Size adjustments needed • Suggestions for larger dimensions • Prevention of piece loss	• "Durability" • "size of the materials and its durability" • "Bigger sizes and more colorful" • "the size so it won't get lost easily"

Note: Themes derived from qualitative analysis of teachers' open-ended responses

Learner Testing Phase

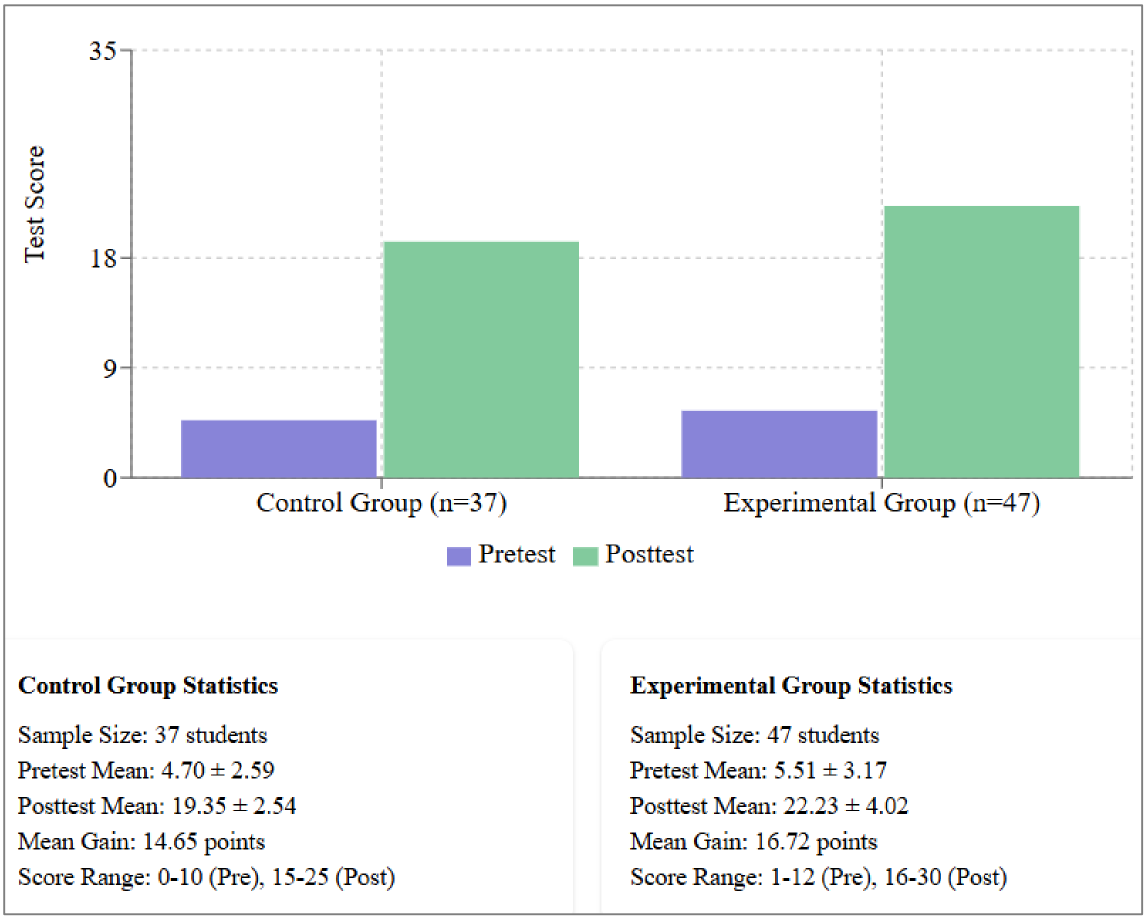
The preliminary testing with 84 grade 7 learners showed promising results. Initially, as shown in Table 5, both the control and experimental groups started at relatively similar levels, with no significant difference at pretest ($U = 692$, $z = -1.60$, $p > 0.05$). Following the intervention, both groups showed significant improvement, but the experimental group demonstrated significantly better performance in the posttest ($U = 456$, $z = -3.73$, $p < 0.001$).

Table 5: Between-Group Comparisons (Mann-Whitney U Test)

Comparison	U-value	z-score	p-value	Interpretation
Pretest	692	-1.60	$p > 0.05$	Not significant; groups started at similar levels
Posttest	456	-3.73	$p < 0.001$	Highly significant difference between groups

As shown in Figure 3, the experimental group achieved a higher posttest mean score of 22.23 (SD = 4.02) compared to the control group's mean of 19.35 (SD = 2.54), with a larger mean gain of 16.72 points versus 14.65 points for the control group.

Figure 3: Before and After Results Comparison



Learner feedback was overwhelmingly positive, with five major themes emerging: Enhanced Learning Experience, Engagement and Enjoyment, Learning Support, Visual Appeal, and Suggestions for Improvement, as shown in Table 6. Learners particularly emphasized improved understanding and ease of learning, active participation, and the fun learning environment created by the manipulatives.

Table 6: Thematic Analysis of Learner Feedback

Major Theme	Subtheme	Supporting Quotes
Enhanced Learning Experience	Understanding and Comprehension	• "I understand the lesson" • "It helps most in understanding fractions in math" • "It's easy to understand" • "I like the math manipulatives because we can understand the solution" • "It explains more than just teachers talking about it"
	Ease of Learning	• "It's easy to use" • "It's easy to solve and answer the questions" • "I can easily learn fast" • "The manipulatives are easy to use"
Engagement and Enjoyment	Active Participation	• "I like to participate in class when we use these math manipulatives" • "I feel excited when I get to work with these math manipulatives"
	Fun Learning Environment	• "I enjoy working with my classmates when we use these manipulatives" • "Combination of playing and learning math" • "It feels like I am building something and turn back to I was a kid" • "Cool and useful" • "Manipulatives are cool"
Learning Support	Instructional Aid	• "It helps us to learn math" • "We can learn more about math" • "It helps me learn in math" • "It helps a lot when teachers discuss" • "It helps me to know better in math"
	Educational Value	• "The math manipulatives improve our knowledge of solving" • "They are amazing and satisfying. They are useful for learning" • "This can help us learn many lessons" • "It can improve the lesson more"
Visual Appeal	Aesthetic Elements	• "The color scheme is quite amazing" • "These math manipulatives make learning interesting"
Suggestions for Improvement	Physical Modifications	• "It will be cooler if it has magnet" • "Bigger sizes"

Classroom observations confirmed these findings, revealing high levels of learner engagement, effective implementation of the manipulatives, and successful progression from concrete to abstract understanding, as shown in Table 7.

Table 7: Thematic Analysis of Classroom Observations

Major Theme	Sub-themes	Key Observations
Physical and Pedagogical Preparedness	• Classroom Organization • Instructional Planning	• Well-organized and accessible materials • Appropriate arrangement for group work • Strategic placement of visual tools • Clear learning objectives • Structured progression of concepts • Integration of manipulatives with curriculum
Effective Teaching Strategies	• Direct Instruction • Conceptual Development	• Clear communication of objectives • Proper demonstration techniques • Guided practice opportunities • Explicit connections between concrete and abstract concepts • Progressive movement from manipulatives to symbolic representation • Multiple strategy approaches
Learner Engagement and Participation	• Active Learning • Collaborative Learning	• Consistent learner enthusiasm • High levels of participation • Initiative in asking questions • Effective group work • Peer interactions • Shared problem-solving experiences
Learning Process Development	• Problem-Solving Skills • Conceptual Understanding	• Active use of manipulatives • Development of multiple strategies • Application of concepts • Clear explanations using materials • Successful symbolic representation • Connection between concrete and abstract concepts
Assessment and Learning Outcomes	• Continuous Assessment • Learning Effectiveness	• Regular formative assessment • Consistent feedback • Monitoring of progress • Enhanced learner learning • Successful concept integration • Demonstrated understanding

5. Conclusion

The development phase of this study has successfully created and validated tangible mathematics materials specifically designed to address the identified needs of grade 7 learners in understanding rational numbers. The systematic design-based research approach resulted in manipulatives that were highly rated by expert evaluators, enthusiastically received by teachers, and demonstrably effective in preliminary learner testing.

The panel evaluation demonstrated exceptional quality and precision, with the materials exceeding minimum requirements across all evaluation areas. The content evaluation yielded an impressive score of 37.8 out of 40 points, the technical accuracy assessment achieved a perfect score of 16 out of 16, and the instructional and technical design evaluation scored 22 out of 24 points.

Teacher feedback highlighted the materials' potential for enhancing learner understanding through visual and hands-on learning experiences, while also providing practical suggestions for effective classroom use. Particularly significant was teachers' recognition of the manipulatives' potential to improve learners' attitudes toward mathematics, especially concerning fractions.

Preliminary testing with 84 grade 7 learners provided strong evidence of the materials' effectiveness, with the experimental group showing significantly better performance than the control group. Both quantitative assessment data and qualitative classroom observations confirmed that the tangible materials successfully enhanced learner understanding, engagement, and problem-solving abilities in rational number operations. These findings provide a strong foundation for the subsequent implementation phase of this research, suggesting that the developed tangible mathematics materials have significant potential to address the persistent challenges in grade 7 mathematics education, particularly in the critical area of rational numbers.

6. Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The tangible mathematics materials were developed solely for educational purposes, and the authors have no commercial interests related to these materials.

7. Acknowledgement

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