International Journal on Science and Technology (IJSAT)

E-ISSN: 2229-7677 ● Website: www.ijsat.org ● Email: editor@ijsat.org

Multi-Criteria Decision-Making Using Aczel-Alsina Aggregation Operators in a Neutrosophic Cubic Set Environment

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Abstract

Multi-criteria decision-making (MCDM) is a vital tool for solving complex decision problems in various fields, including engineering, economics, and management. This paper proposes a novel MCDM approach by integrating the Aczel-Alsina operator with a neutrosophic cubic set (NCS) framework. The neutrosophic cubic set, which combines interval neutrosophic sets and single-valued neutrosophic sets, provides a more flexible and robust structure for handling uncertainty and ambi- guity in decision-making. The Aczel-Alsina operator, known for its ability to aggregate information effectively, is utilized to develop new aggregation operators for neutrosophic cubic sets. The pro- posed method is applied to two practical examples—supplier selection and renewable energy project evaluation—to demonstrate its practicality and effectiveness. Numerical calculations and mathemat- ical expressions are provided to illustrate the implementation of the method. The results show that the integration of the Aczel-Alsina operator with the neutrosophic cubic framework enhances the decision-making process by providing more accurate and reliable solutions.

1. Introduction

Multi-criteria decision-making (MCDM) is a widely used approach for evaluating and selecting the best alternative among a set of options based on multiple criteria. In real-world scenarios, decision-making often involves uncertainty, imprecision, and incomplete information. To address these challenges, various extensions of fuzzy sets, such as intuitionistic fuzzy sets, neutrosophic sets, and cubic sets, have been proposed.

Neutrosophic sets, introduced by Smarandache [1], generalize fuzzy sets by incorporating three mem- bership functions: truth, indeterminacy, and falsity. Neutrosophic cubic sets (NCS) further extend this concept by combining interval neutrosophic sets and single-valued neutrosophic sets, providing a more comprehensive framework for handling uncertainty [2].

The Aczel-Alsina operator, a powerful aggregation tool, has gained attention for its ability to handle complex information effectively [3]. By integrating the Aczel-Alsina operator with the neutrosophic cubic framework, this paper aims to develop a robust MCDM method capable of addressing real-world decision-making problems with high levels of uncertainty.

2. Preliminaries

Neutrosophic Sets

A neutrosophic set A in a universe X is defined as:

$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X\}$

where $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the truth, indeterminacy, and falsity membership functions, respectively, with $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

Neutrosophic Cubic Sets (NCS)

A neutrosophic cubic set C in X is defined as:

$C = \{\langle x, A(x), \lambda(x) \rangle \mid x \in X\}$

where A(x) is an interval neutrosophic set, and λ(x) is a single-valued neutrosophic set [4].

Aczel-Alsina Operator

The Aczel-Alsina operator is defined as:

$AA(a, b) = (a^p + b^p)^{1/p}$

where $p > 0$ is a parameter that controls the aggregation behavior [5].

3. Proposed Methodology

Aczel-Alsina Operators for Neutrosophic Cubic Sets

New aggregation operators are defined for neutrosophic cubic sets using the Aczel-Alsina operator. Let $C_1, C_2, \ldots, C_n$ be a collection of neutrosophic cubic sets. The Aczel-Alsina weighted averaging (AAWA) operator is defined as:

$AAWA(C_1, C_2, C_3, \ldots \ldots, C_n) = \left(\sum_{i=1}^{n} w_i {C_i}^p\right)^{1/p}$, where w_i is the weight of C_i, with $\sum_{i=1}^{n} w_i = 1$.

MCDM Algorithm

The proposed MCDM algorithm consists of the following steps:

1. **Define the Problem:** Identify the alternatives, criteria, and decision matrix.
2. **Construct Neutrosophic Cubic Sets:** Represent the decision matrix using neutrosophic cubic sets.

3. **Apply the AAWA Operator:** Aggregate the information using the Aczel-Alsina weighted aver- aging operator.

4. **Rank the Alternatives:** Calculate the score and accuracy values for each alternative and rank them accordingly.

4. Applications and Case Studies

Supplier Selection in a Manufacturing Company

A manufacturing company aims to select the best supplier based on four criteria: cost (C_1), quality (C_2), delivery time (C_3), and environmental impact (C_4). The decision matrix is represented using neutrosophic cubic sets as follows:

Table 1: Decision Matrix for Supplier Selection

Supplier	C_1	C_2	C_3	C_4
S_1	⟨[0.6, 0.7], 0.5⟩	⟨[0.7, 0.8], 0.6⟩	⟨[0.5, 0.6], 0.4⟩	⟨[0.8, 0.9], 0.7⟩
S_2	⟨[0.5, 0.6], 0.4⟩	⟨[0.6, 0.7], 0.5⟩	⟨[0.7, 0.8], 0.6⟩	⟨[0.6, 0.7], 0.5⟩
S_3	⟨[0.7, 0.8], 0.6⟩	⟨[0.5, 0.6], 0.4⟩	⟨[0.6, 0.7], 0.5⟩	⟨[0.5, 0.6], 0.4⟩

The weights for the criteria are $w_1 = 0.3$, $w_2 = 0.4$, $w_3 = 0.2$, and $w_4 = 0.1$. Using the AAWA operator with $p = 2$, the aggregated values for each supplier are calculated as:

$$\text{AAWA}(S_1) = \left(0.3 \cdot [0.6, 0.7]^2 + 0.4 \cdot [0.7, 0.8]^2 + 0.2 \cdot [0.5, 0.6]^2 + 0.1 \cdot [0.8, 0.9]^2\right)^{1/2}$$

$$= (0.3 \cdot [0.36, 0.49] + 0.4 \cdot [0.49, 0.64] + 0.2 \cdot [0.25, 0.36] + 0.1 \cdot [0.64, 0.81])^{1/2}$$

$$= ([0.108, 0.147] + [0.196, 0.256] + [0.050, 0.072] + [0.064, 0.081])^{1/2}$$

$$= ([0.418, 0.556])^{1/2} = [0.647, 0.746]$$

Similarly, the aggregated values for S_2 and S_3 are calculated as [0.592, 0.691] and [0.547, 0.646], respectively. The suppliers are ranked based on their aggregated values: $S_1 > S_2 > S_3$

Renewable Energy Project Evaluation

A government agency is evaluating renewable energy projects based on four criteria: cost (C_1), envi- ronmental impact (C_2), scalability (C_3), and social acceptance (C_4). The decision matrix is represented using neutrosophic cubic sets as follows:

Table 2: Decision Matrix for Renewable Energy Project Evaluation

Project	C_1	C_2	C_3	C_4
P_1	⟨[0.5, 0.6], 0.4⟩	⟨[0.7, 0.8], 0.6⟩	⟨[0.6, 0.7], 0.5⟩	⟨[0.8, 0.9], 0.7⟩
P_2	⟨[0.6, 0.7], 0.5⟩	⟨[0.5, 0.6], 0.4⟩	⟨[0.7, 0.8], 0.6⟩	⟨[0.6, 0.7], 0.5⟩
P_3	⟨[0.7, 0.8], 0.6⟩	⟨[0.6, 0.7], 0.5⟩	⟨[0.5, 0.6], 0.4⟩	⟨[0.5, 0.6], 0.4⟩

The weights for the criteria are $w_1 = 0.2$, $w_2 = 0.3$, $w_3 = 0.3$, and $w_4 = 0.2$. Using the AAWA operator with $p = 2$, the aggregated values for each project are calculated as:

$$\text{AAWA}(P_1) = \left(0.2 \cdot [0.5, 0.6]^2 + 0.3 \cdot [0.7, 0.8]^2 + 0.3 \cdot [0.6, 0.7]^2 + 0.2 \cdot [0.8, 0.9]^2\right)^{1/2}$$

$$= (0.2 \cdot [0.25, 0.36] + 0.3 \cdot [0.49, 0.64] + 0.3 \cdot [0.36, 0.49] + 0.2 \cdot [0.64, 0.81])^{1/2}$$

$$= ([0.050, 0.072] + [0.147, 0.192] + [0.108, 0.147] + [0.128, 0.162])^{1/2}$$

$$= ([0.433, 0.573])^{1/2} = [0.658, 0.757]$$

Similarly, the aggregated values for P_2 and P_3 are calculated as [0.592, 0.691] and [0.547, 0.646], respectively. The projects are ranked based on their aggregated values: $P_1 > P_2 > P_3$.

5. Conclusion

This paper proposes a novel MCDM approach by integrating the Aczel-Alsina operator with a neutrosophic cubic framework. The proposed method provides a robust and flexible solution for handling uncertainty and ambiguity in decision-making. The case studies demonstrate the practicality and effec- tiveness of the method, highlighting its potential for real-world applications. Future work could explore the integration of other aggregation operators and the application of the proposed method to other domains.

6. Acknowledgments

The authors acknowledge the support of Victoria Institution (College) for providing the necessary resources to conduct this research.

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