International Journal on Science and Technology (IJSAT)
E-ISSN: 2229-7677 ● Website: www.ijsat.org ● Email: editor@ijsat.org

Mean Cordial Labeling Patterns in Shadow-Graphs of Paths

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Abstract

Let f be a function from $V\ (G)$ to $\{0, 1, 2\}$. Give a numerical value (label) from {0, 1, 2} for every edgeuv. f is called a mean cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1, \forall i, j \in \{0, 1, 2\}$ where $v_f(x)$and $e_f(x)$represent the number of vertices and edges respectively labeled as$x(x = 0, 1, 2)$. A graph with mean cordial labeling is called a mean cordial graph. In this paper, we study the mean cordial labeling pattern of shadow-graphs of path graphs$D_2(P_n)$,for $n \geq 2$

Keywords: Mean Cordial Labeling, Shadowgraph, Path Graph

Subject Classification: 05C78

1. Introduction:

The graphs discussed are simple, finite, and undirected. For a graph GG, V(G)V(G) denotes the set of vertices, and E(G)E(G) denotes the set of edges. The *order* of GG refers to the number of vertices in V(G)V(G), while the *size* refers to the number of edges in E(G)E(G). Labeled graphs find applications in various fields, including tracking systems, route design, broadband networks, astrographs, and coding/encoding processes[1].The concept of cordial labeling was introduced by Cahit in 1987[2].

1.1 Definition:

Let $f : V\ (G) \rightarrow \{0,1\}$ be a mapping. Give a numerical value (label) $|f(u) - f(v)|$for every edgeuv. f is called a cordial labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1,\ \forall i, j \in \{0,1\}$ where $v_f(x)$and $e_f(x)$represent the number of vertices and edges respectively labeled as$x(x = 0,1)$. A graph that admits cordial labeling is called a cordial graph [2].

1.2 Definition:

Let $f : V\ (G) \rightarrow \{0,1,2\}$ be a function.Give a numerical value (label)$\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$for every edge uv.f is called a mean cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$and $|e_f(i) - e_f(j)| \leq 1,\ \forall i, j \in \{0,1,2\}$ where $v_f(x)$and $e_f(x)$represent the number of vertices and edges respectively labeled as$x(x = 0,1,2)$. A graph that admits a mean cordial labeling is called a mean cordial graph[3].

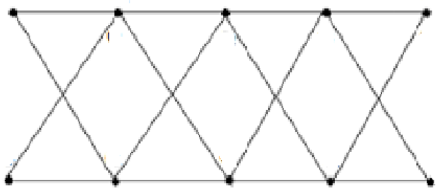
If we reduce the codomain of f to $\{0,1\}$ then this definition becomes the definition of product cordial labeling.M. Sundaram, R. Ponraj, and Somasundaram established the concept of cordial product labelling [4].

Iexamine the mean cordial labeling pattern of shadow graph of path graphs. The sign$\lfloor x \rfloor$represents the greatest integer not exceeding$x$ and $\lceil x \rceil$represents the smallest integer exceeding$x$. Terminologies not described in this paper are used in Harary's context[2].

1.3 Definition:
The path graph P_n is a tree with two vertices of degree1, and the other $n-2$ vertices of degree 2. A path graph is therefore a graph that can be drawn so that all of its vertices and edges lie on a single straight line [5]**.**

1.4 Definition:
The shadow-graph $D2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in $G'$to the neighbours of the corresponding vertex v'in G''[6].
Example1:

$u_1 u_2 u_3 u_4 u_5$



$v_1 v_2 v_3 v_4 v_5$

Fig.1 $D_2(P_5)$

Here$|V(D_2(P_n))| = 2n$ and $|E(D_2(P_n))| = 4(n-1)$

1.5Definition:

Let f be a mapping from $V(G)$ to $\{0,1,2\}$. For every edge uv of G, nominate the label$\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ or$\left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$. f is called a mean cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$, $\forall i,j \in \{0,1,2\}$ where $v_f(x)$and $e_f(x)$represent the number of vertices and edges respectively labeled as$x (x = 0,1,2)$. An illustration using mean cordial labeling is referred to as a mean cordial graph.

2. Main Result:
Theorem: Let P_n be a path graph on n vertices, then $D_2(P_n)$admits mean cordial labeling for $n \geq 2$.
Proof. Define $f : V(G) \rightarrow \{0,1,2\}$ *and* $g : E(G) \rightarrow \{0,1,2\}$.
Case 1: $n \equiv 0 (mod3)$: That is, $n = 3k$, where k is a positive integer.

We define vertex labeling as follows: $f(u_i) = \begin{cases} 0 & i \equiv 1\ (mod3) \\ 2 & i \equiv 2\ (mod3) \\ 1 & i \equiv 0\ (mod3) \end{cases}$

$$f(v_i) = \begin{cases} 2 & i \equiv 1\ (mod3) \\ 0 & i \equiv 2\ (mod3) \\ 1 & i \equiv 0\ (mod3) \end{cases}$$

We define edge labeling as follows:

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$g\,(u_i\,v_{i-1}) \;=\; \left\lfloor\frac{f(u_i)+f(v_{i-1})}{2}\right\rfloor \qquad 2 \le i \le n,$

$g\,(u_i\,v_{i+1}) \;=\; \left\lceil\frac{f(u_i)+f(v_{i+1})}{2}\right\rceil \qquad 1 \le i \le n-1$ and

$g\,(u_i\,u_{i+1}) \;=\; \left\lceil\frac{f(u_i)+f(u_{i+1})}{2}\right\rceil \qquad 1 \le i \le n-1.$

$g\,(v_i\,v_{i+1}) = \left\lfloor\frac{f(v_i)+f(v_{i+1})}{2}\right\rfloor \qquad 1 \le i \le n-1$

In this case, we have $v_f(0) = v_f(1) = v_f(2) = 2k$, and

$e_g(0) = e_g(2) = 4k-1, e_g(1) = 4k-2.$

Example: MCL of $D_2(P_3)$ is shown in Fig.2

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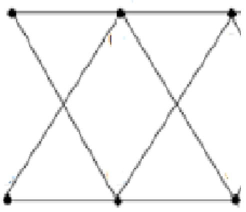

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Fig.2 $D_2(P_3)$

Case 2: $n \equiv 1(mod3)$: That is, $n = 3k+1$, where k is a positive integer.

We define vertex labeling as follows: $f\,(u_i) = \begin{cases} 0 & i \equiv 1\,(mod3),\ i < n \\ 2 & i \equiv 2\ (mod3) \\ 1 & i \equiv 0\ (mod3) \end{cases}$

$$f(u_{3k+1}) = 2$$

$$f\,(v_i) = \begin{cases} 2 & i \equiv 1\,(mod3), i < n \\ 0 & i \equiv 2\ (mod3) \\ 1 & i \equiv 0\ (mod3) \end{cases}$$

$$f(v_{3k+1}) = 0$$

We define edge labeling as follows:

$g\,(u_i\,v_{i-1}) \;=\; \left\lfloor\frac{f(u_i)+f(v_{i-1})}{2}\right\rfloor \qquad 2 \le i \le n,$

$g\,(u_i\,v_{i+1}) \;=\; \left\lceil\frac{f(u_i)+f(v_{i+1})}{2}\right\rceil \qquad 1 \le i \le n-1$ and

$g\,(u_i\,u_{i+1}) \;=\; \left\lceil\frac{f(u_i)+f(u_{i+1})}{2}\right\rceil \qquad 1 \le i \le n-1.$

$$g\ (v_i\, v_{i+1}) = \left\lfloor \frac{f(v_i) + f(v_{i+1})}{2} \right\rfloor \qquad 1 \le i \le n-1$$

In this case, we have $v_f(0) = v_f(2) = 2k + 1 v_f(1) = 2k$, and

$e_g(0) = e_g(2) = e_g(1) = 4k.$

Example: MCL of $D_2(P_4)$ is shown in Fig.3

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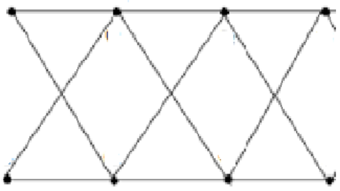

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Fig.3$D_2(P_4)$

Case 3: $n \equiv 2(mod3)$: That is, $n = 3k + 2$, where k is a positive integer.

We define vertex labeling as follows: $f\ (u_i) = \begin{cases} 0 & i \equiv 1\ (mod3), i < n-1 \\ 2 & i \equiv 2\ (mod3), i < n \\ 1 & i \equiv 0\ (mod3) \end{cases}$

$$f(u_{3k+1}) = 2$$
$$f(u_{3k+2}) = 1$$

$$f\ (v_i) = \begin{cases} 2 & i \equiv 1\ (mod3), i < n-1 \\ 0 & i \equiv 2\ (mod3), i < n \\ 1 & i \equiv 0\ (mod3) \end{cases}$$

$$f(v_{3k+1}) = 0$$
$$f(v_{3k+1}) = 0$$

We define edge labeling as follows:

$$g\ (u_i\, v_{i-1}) = \left\lfloor \frac{f(u_i)+f(v_{i-1})}{2} \right\rfloor \qquad 2 \le i \le n,$$

$$g\ (u_i\, v_{i+1}) = \left\lceil \frac{f(u_i)+f(v_{i+1})}{2} \right\rceil \qquad 1 \le i \le n-1 \quad \text{and}$$

$$g\ (u_i\, u_{i+1}) = \left\lceil \frac{f(u_i)+f(u_{i+1})}{2} \right\rceil \qquad 1 \le i \le n-1.$$

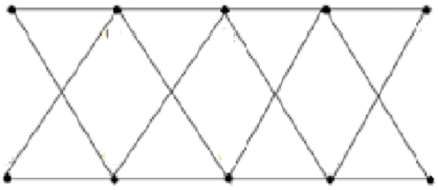
$$g\ (v_i\, v_{i+1}) = \left\lfloor \frac{f(v_i) + f(v_{i+1})}{2} \right\rfloor \qquad 1 \le i \le n-1$$

In this case, we have $v_f(0) = 2k + 2, v_f(1) = v_f(2) = 2k + 1$, and

$e_g(0) = 4k + 2, e_g(1) = e_g(2) = 4k + 1.$

Example: MCL of $D_2(P_5)$ is shown in Fig.4

0 2 1 2 1



2 0 1 0 0

Fig.4 $D_2(P_5)$

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