

Hermitian Matrix and Skew Hermitian Matrix

M Vinothkumar Muniyandi

Lecturer

Introduction:

Hermitian matrix and skew hermitian matrix in the same condition satisfying by, $A=A^T, A=-A^T$.

Hermitian matrix:

Definition: A hermitian matrix is a square matrix that is equal to its conjugate transpose in other words take the complex conjugate of each element the transpose the matrix you get the matrix back, $A=A^T$.

Skew hermitian matrix:

Definition: Hermitian matrix is square matrix that A is said to be skew hermitian e if it's conjugate transpose is equal to its negative , mathematical that expressed as $A=-A^T$.

Hermitian matrix and skew hermitian matrix example:

$$\text{i) } A = \begin{bmatrix} 3 & 1-i \\ 1+i & 2 \end{bmatrix}$$

$$= 6 - (1+i)(1-i) = 6 - (1-i+i-i^2) = 6 - 1 + 1 = 6.$$

$$A^T = \begin{bmatrix} 3 & 1+i \\ 1+i & 2 \end{bmatrix}$$

$$= 6 - (1-i+i-i^2) = 6 - 1 + 1 = 6.$$

$A=A^T$ is a Hermitian matrix .

Skew hermitian matrix example:

$$\text{i) } A = \begin{bmatrix} 1 & 3 \\ 2 & 2i \end{bmatrix}$$

$$= 2i(i) - 3(2) = 2(-1) - 6 = -8.$$

$$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 2i \end{bmatrix}$$

$$2 \text{ & } 2i$$

$$\end{bmatrix}$$

$$= 2i^2 - 3(2) = 2(-1) - 6 = -8.$$

$A = -A^T$. It is a skew hermitian matrix.

Even odd number of matrix:

Definition: Even odd number of matrix first row is a even number and second row is a odd number is a called by positive value is a even odd number of matrix is a $A = A^T$.

Odd even number of matrix:

Definition: Odd even number of matrix is first row is a odd number and second row is a even number is a called negative value is $A = -A^T$.

Even odd and odd even number of matrix example:

i) $A = \begin{bmatrix}$

$$2 \text{ & } 4 \\$$

$$1 \text{ & } 3$$

$$\end{bmatrix}$$

$$= 6 - 4 = 2.$$

$$A^T = \begin{bmatrix}$$

$$2 \text{ & } 1 \\$$

$$4 \text{ & } 3$$

$$\end{bmatrix}$$

$$= 6 - 4 = 2. A = A^T. \text{ Is a even odd number of matrix.}$$

ii) $A = \begin{bmatrix} 1 \text{ & } 3 \\$

$$2 \text{ & } 4$$

$$\end{bmatrix}$$

$$= 4 - 6 = -2.$$

$$= 4 - 6 = -2.$$

ii) $A^T = \begin{bmatrix} 1 \text{ & } 2 \\$

$$3 \text{ & } 4$$

$\end{bmatrix}$

$n=4-6=-2$. $A=-A^T$ is a odd even number of matrix.

Conclusion:

Hermitian matrix and skew hermitian matrix is a equal condition of the satisfy by the even odd and odd even number of matrix is a called by “equivalence matrix”. $A=A^T$ and $A=-A^T$.

Book reference:

Archibald, Tom (2010-12-31), Gowers, Timothy; Barrow-Green, June; Leader, Imre (eds.), “VI.47 Charles Hermite”, The Princeton Companion to Mathematics, Princeton University Press, p. 773, doi:10.1515/9781400830398.773a, ISBN 978-1-4008-3039-8, retrieved 2023-11-15[^] Ribeiro, Alejandro. “Signal and Information Processing” (PDF).[^] “MULTIVARIATE NORMAL DISTRIBUTIONS” (PDF).[^] Lau, Ivan. “Hermitian Spectral Theory of Mixed Graphs” (PDF).[^] Liu, Jianxi; Li, Xueliang (February 2015). “Hermitian-adjacency matrices and Hermitian energies of mixed graphs”. Linear Algebra and Its Applications. 466: 182–207. Doi:10.1016/j.laa.2014.10.028.[^] Frankel, Theodore (2004). The Geometry of Physics: an introduction. Cambridge University Press. P. 652. ISBN 0-521-53927-7.[^] Physics 125 Course Notes Archived 2022-03-07 at the Wayback Machine at California Institute of Technology[^] Trefethan, Lloyd N.; Bau, III, David (1997). Numerical linear algebra. Philadelphia, PA, USA: SIAM. P. 34. ISBN 0-89871-361-7. OCLC 1348374386.[^] Jump up to:^a b c Horn, Roger A.; Johnson, Charles R. (2013). Matrix Analysis, second edition. Cambridge University Press. ISBN 9780521839402.[^] Also known as the Rayleigh–Ritz ratio; named after Walther Ritz and Lord Rayleigh.[^] Parlet B. N. The symmetric eigenvalue problem, SIAM, Classics in Applied Mathematics, 1998