International Journal on Science and Technology (IJSAT)

E-ISSN: 2229-7677 ● Website: www.ijsat.org ● Email: editor@ijsat.org

Mathematical Model of Bingham Fluid inside Asymmetric Stenosed Artery and Its Applications

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Abstract

This study is to analysis the two-layered model of blood flow inside asymmetric stenosed artery with the consideration that blood behaves like a Newtonian and non-Newtonian fluid with different viscosities in the Peripheral layer and core regions. Further, in view of the existence of a velocity slip in blood flow regime, an axial slip condition as reported by many researchers is introduced at interface of the two-layers in the present analysis. The expressions for the flow characteristics namely the velocity profile, flow rate , the resistance to flow, the wall shear stress at the stenosis throat within the tube are obtained. Discussions are done from a physiological point of view with the help of graph.

Keywords: Fluid, Viscosities, resistance to flow, shear stress.

Introduction

The blood vessels carry blood from the heart to all the organs and tissues of the body including brain, kidneys, gut, muscles and heart itself. According to reports, atherosclerosis is one of the most dangerous cardiovascular (cvs) illnesses and one of the leading causes of death in developed nations (Caro, 1981). Atherosclerosis or stenosis, which can develop symmetrically or asymmetrically, progresses along the innermost arterial wall in three stages: (i) the initial stage, during which the first thickening starts; (ii) the growing stage generally; and (iii) the severe stage, during which the blood supply to the heart muscle iscompromised.Stenosis,whichoftendevelops as a resultof an irregular growth at the artery lumen, is no w recognised as a cardiac condition that is rapidly spreading among people of all ages.Another form of v ascular disease is stenosis, which has been linked to changes in blood flow, flow patterns, pressure drop, and flow impedance (Liu et al., 2004; Poltem et al., 2006According to numerous studies (Young and Tsai, 1973; Caro, 1981; Mates et al., 1978), hydrodynamic variables may significantly contribute to the creation, development, and advancement of an arterial stenosis. Additionally, it is stated that the fluid dynamics and rheologic characteristics of blood and blood flow may be crucial in the basic knowledge, identification, and management of a number of cardiovascular and arterial disorders (Chien, 1981; Light hill, 1975; Theoretical efforts (Vand, 1948; Bloch, 1962; Brunn, 1975; Nubar, 1967) and experimental studies (Bugliarello and Hayden, 1962; Bennet, 1967) are both included in the investigations that have improved our understanding of the flow disruptions caused by a stenosis. By using protochromic tracer methods, distinct velocity profiles for pulsatile flow through tubes with constrictions of varying symmetry and degree of obstruction are achieved in those investigations (Ojha et al., 1989). Smith (1979) has conducted extensive studies of steady flows through an axi-symmetric stenosed artery by an analytical approach. The study has revealed that the resulting flow patterns are highly dependent on geometry of the stenosis and the overall Reynolds number of the flow. Deshpande et al. (1976) have employed the finite difference scheme to consider the flow situation through axi-symmetric and asymmetric conditions and with a range of Reynolds number from laminar to turbulent flows. It is seen that all these factors have considered significantly to the flow behaviour. Atherosclerosis changes this flowpattern and forces the flow to produce highly disturbed flow which can be aggravated by other diseased conditions that may alter artery flow or blood properties. Mukhopadhyaya and Layek (2008) have presented a symmetric analysis of flow behaviour in a two-dimensional tube model as an artery with a locally variable shapedconstriction. Srivastav et.al (2016) has presented a mathematical model for analysis of blood flow in a stenosed artery with perimable wall and they discussed the variation of variation of different flow variables from a physiological point of view.

Bingham plastic models are examined by several authors (MohanRao, 1965; Kapur and Dikshit, 1965). Steady flow of Bingham plastic fluid through a circular tube subject to a pressure gradient and the usual zero-slip condition (Schlichting, 1968) at boundary, is investigated in the models (Fung, 1981). Biswas and Bhattacherjee (2003) have modeled the steady annular flow of Bingham fluid in a catheterized stenosed artery. Biswas and Chakraborty (2010) have studied a two-layered Bingham plastic fluid in a stenosed artery with a velocity slip condition at the boundary wall. Bugliarello and Sevilla (1970) and Cokelet (1972) have reported experimentally that for blood flow through small arteries, there exists a cell poor peripheral layer of plasma and a core region of mostly red cells. Misra and Paul (1999) presented a laminar, pulsatile flow model of blood under the influence of an externally imposed body acceleration. In this model, blood is assumed to behave like a Bingham plastic fluid.

In all the above mentioned studies, traditional no-slip boundary condition has been employed. However, a number of studies of suspensions in general and blood flow in particular, both theoretical (Vand, 1948; Nubar, 1967; Sarkar and Jayaraman, 1997 and experimental (Bugliarello, G. and Hayden, 1962; Bennet, 1967) have suggested the likely presence of slip at the boundaries. It seems that consideration of a velocity slip at the vessel wall be quite rational in blood flow modeling.

In view of the above considerations, we are interested to study the two-layered model of blood flow inside asymmetric stenosed artery with the consideration that blood behaves as a Newtonian and non-Newtonian fluid with different viscosities in the ppl and core regions. Further, in view of the existence of a velocity slip in blood flow regime, an axial slip condition as reported by many researchers is introduced at interface of the two-layers, in the present analysis.

Mathematical Formulation

We consider the steady, laminar, and fully developed flow of blood (assumed to be incompressible) in the axial (z) direction through an artery with an axially non- symmetrical but radially symmetrical stenosis at vessel wall. The artery length is assumed to be large enough as compared to its radius so that the entrance, exit and special wall effects can be neglected. The model basically consists of- a core of red blood cell suspension in the middle layer and the peripheral plasma layer in the outer layer (as shown in Fig. 1).

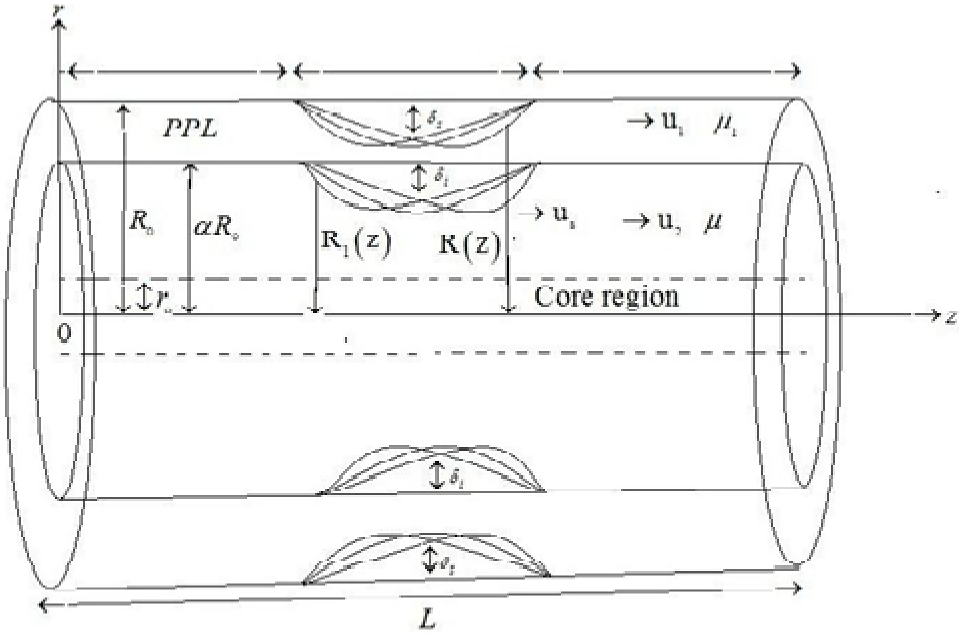

Fig.1:Schematic diagram of Two-Layered Bingham Plastic Model of Blood Flow inside Asymmetric Stenosed Artery

It is assumed that the rheology of blood in the core region has been characterized as a non-Newtonian fluid, obeying the law of Bingham Plastic fluid model and the PPL as a Newtonian fluid with different viscosities μ and μ_1 respectively.

The governing equation of the fluid flow has the usual form

$$c+\frac{\mu_1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_1}{\partial r}\right)=0,\ \text{where c=-}\frac{dp}{dz} \qquad (2.1)$$

The constitutive equation for Bingham plastic fluid is in the form

$$\tau=\pm\tau_0+\mu\dot{e} \qquad (2.2)$$

where τ, τ_0, μ and $\dot{e}$ are the shear stress, yield stress value, viscosity of the fluid and strain rate respectively.

In case of Bingham plastic, it is noticed that the fluid does not flow below the yield stress value τ_0 and above this yield value the fluid flow is possible, so the rise in shear stress is proportional to the shear rate and hence equation (2.2) can be put in the form $\dot{e}=\frac{1}{\mu}\left(\tau_{rz}-\tau_0\right), \tau_{rz}\geq\tau_0$ (2.3)

$$=0, \tau_{rz}\leq\tau_0 \qquad (2.4)$$

In order to obtain the strain rate ($\dot{e}$) of the fluid in the artery wall, we integrate the equation (2.1) twice and we get $u_1(r) = -\dfrac{cr^2}{4\mu_1} + A\ln r + B$ (2.5)

Where A and B are constants of integration and these constants are to be determined with the help of usual conditions:

For symmetry condition at r=0 and zero slip at r= R(z), the equation (2.5) becomes

$$u(r) = \frac{c}{4\mu}(R^2 - r^2), 0 \le r \le r(z) \qquad (2.6)$$

Now the shear stress component at any distance 'r' from the tube axis is given by (Schlichting, 1968)

$$\tau_{rz} = \mu\frac{\partial u}{\partial r} = \mu\dot{e} \qquad (2.7)$$

With the help of equation(2.6) the equation (2.7) becomes

$$\tau_{rz} = -\frac{c}{2}r = \frac{r}{2}\frac{dp}{dz} \qquad (2.8)$$

Which implies τ_{rz} is proportional to r where $\frac{1}{2}\frac{dp}{dz}$ is a constant of proportionality.

The expression for wall shear stress (τ_{rz}) is obtained from equation (2.8) as

$$\tau_w = \tau_{rz}|_{r=R} = \mu\frac{\partial u}{\partial r}|_{r=R} = -\frac{c}{2}R = \frac{R}{2}\left(\frac{dp}{dz}\right) \qquad (2.9)$$

Using equation (2.8), we can get yield stress

$$\tau_0 = \tau_{rz}|_{r=r_0} = \mu\frac{\partial u}{\partial r}|_{r=r_0} = -\frac{c}{2}r_0 = \frac{r_0}{2}\left(\frac{dp}{dz}\right) \qquad (2.10)$$

In between two stresses τ_w and τ_0, there arise two cases viz.

I. If the value of yield stress is smaller than the shear stress i.e. $\tau_0 < \tau_w$, then there will be flow and for that u=u(r) when $\frac{dp}{dz} > \frac{2}{R}\tau_0$ (2.11)

II. If the value of yield stress is greater than the shear stress i.e . $\tau_0 > \tau_w$, no flow will be possible then u(r) =0, when $\frac{dp}{dz} < \frac{2}{R}\tau_0$

So the Bingham equations (2.3) and (2.4) can be written in the form

$$\dot{e} = \frac{1}{\mu}(\tau_w - \tau_0), \tau_w \ge \tau_0 \qquad (2.12)$$

$$=0, \tau_w \le \tau_0 \qquad (2.13)$$

From equation (2.13), we have seen that the strain rate vanishes i.e. $\dot{e}$=0 which implies that $\frac{\partial u}{\partial r}$=0 (2.14)

Which on integration, we get u(r) = constant =u_0 , when $\tau_w = \tau_0$ where u_0 is the core velocity at r = r_0 . Thus for blood flow, there arises three regions viz. $0 \le r \le r_0$, $r_0 \le r \le R_1$ and $R_1 \le r \le R$ indicating that velocity profile will be flat for the region $0 \le r \le r_0$ and for the region $r_0 \le r \le R_1$, velocity profile will exhibit deviation from the flat profile, hence Bingham equation (2.8) has to be applied for this region of blood flow (Fung, 1981). As a consequence of above considerations the following equations are made. Using equation (2.8) and (2.10), the equation (2.3) becomes

$$\frac{du_2}{dr} = \frac{c}{2\mu}(r_0 - r), r_0 \le r \le R_1 \qquad (2.14)$$

On integrating we get $u_2 = \frac{cr}{4\mu}(2r_0 - r) + A_2$ (2.15)

The equation (2.5) can be written as for Peripheral layer

$$u_1 = \frac{cr}{4\mu_1} + A_1 \ln r + B_1, R_1 \le r \le R \qquad (2.15)$$

Also the equation (2.4) can be written as U_0 = constant $= A_3$

Where A_1, A_2, A_3 and B_1 are constants which are to be determined with the following boundary conditions:

1. $u_1=0$, at $r=R$ (2.16)
2. $u_2 - u_1 = u_s$, at $r=R_1$ (2.17)
3. $u_2 = u_0 = A_3$, at $r=r_0$ (2.18)
4. u_1 is finite at the axis (2.19)

3. Solution of the problem:

Applying conditions (2.16) and (2.19) to the equation (2.15), we the expression for velocity in the Peripheral layer

$$u_1 = \frac{c}{4\mu_1}(R^2 - r^2), R_1 \le r \le R \qquad (3.1)$$

By the condition (2.17), we get the expression for velocity in the core region

$$u_2 = u_s + \frac{c}{4\mu_1}(R^2 - R_1^2) + \frac{c}{4\mu}(R_1 - r)(R_1 + r - 2r_0), r_0 \le r \le R_1 \qquad (3.2)$$

Also from the condition (2.18), we have

$$u_0 = u_s + \frac{c}{4\mu_1}(R^2 - R_1^2) + \frac{c}{4\mu}(R_1 - r_0)^2, 0 \le r \le r_0 \qquad (3.3)$$

The volumetric flow rate Q can be obtained by the formula

$$Q = 2\pi\left[\int_{r=0}^{r_0} ru_0 dr + \int_{r_0}^{R_1} ru_2 dr + \int_{R_1}^{R} ru_1 dr\right] \qquad (3.4)$$

$$= Q_1 + Q_2 + Q_3$$

Where $Q_1 = \int_{r=0}^{r_0} ru_0 dr$, $Q_2 = \int_{r_0}^{R_1} ru_2 dr, Q_3 = \int_{R_1}^{R} ru_1 dr$ which can be expressed by using the equation (3.3) as follows:

$$Q = \pi R_1^2 u_s + \frac{\pi c}{8\mu_1} R^4 \left\{1 - \left(\frac{R_1}{R}\right)^4\right\} + \frac{\pi c R_1^4}{8\mu} \varnothing(\alpha) \qquad (3.5)$$

Where $\varnothing(\alpha) = 1 - \frac{4}{3}\alpha + \frac{1}{3}\alpha^4$ (3.6)

The non-dimensional form of flow variables for peripheral layer

$$\overline{u_1} = \overline{R^2}\left\{1 - \left(\frac{r}{R}\right)^2\right\}, \frac{R_1}{R} \le \frac{r}{R} \le 1 \quad (3.6)$$

Flow variables for core region

$$\overline{u_2} = \overline{u_s} + \frac{\mu_1}{\mu}\left\{\overline{R_1} - \overline{R}\left(\frac{r}{R}\right)\right\}\left\{\overline{R_1} + \overline{R}\left(\frac{r}{R}\right) - 2\overline{r_0}\right\} + \left(\overline{R}^2 - \overline{R_1}^2\right),\ \frac{r_0}{R} \le \frac{r}{R} \le \frac{R_1}{R} \quad (3.7)$$

Also from equation (3.3) we have

$$\overline{u_0} = \overline{u_s} + \frac{\mu_1}{\mu}\left(\overline{R_1^2} - \overline{r_0^2}\right)^2, 0 \le \frac{r}{R} \le \frac{r_0}{R} \quad (3.8)$$

Again a second representation of flow rate (Q) is given by the following formula

$$\overline{Q} = \frac{Q}{Q_0} \text{ where } Q_0 = \frac{\pi c R_0^4}{8\mu_1}$$

From equation (3.5)

$$\overline{Q} = 2\overline{R_1^2}\,\overline{u_s} + \left[\overline{R}^4 - \left(1 - \frac{\mu_1}{\mu}\varnothing\left(\overline{\alpha}\right)\right)\overline{R_1}^4\right] \quad (3.9)$$

Pressure gradient:

$$\overline{\left(\frac{dp}{dz}\right)} = \left[\overline{R^4} - \left(1 - \frac{\mu_1}{\mu}\phi\left(\overline{\alpha}\right)\overline{R_1^4}\right)\right]^{-1}\left(\overline{Q} - 2\overline{R^2}\,\overline{u_s}\right) = \overline{\left(\frac{dp}{dz}\right)}_{av} \quad (3.1)$$

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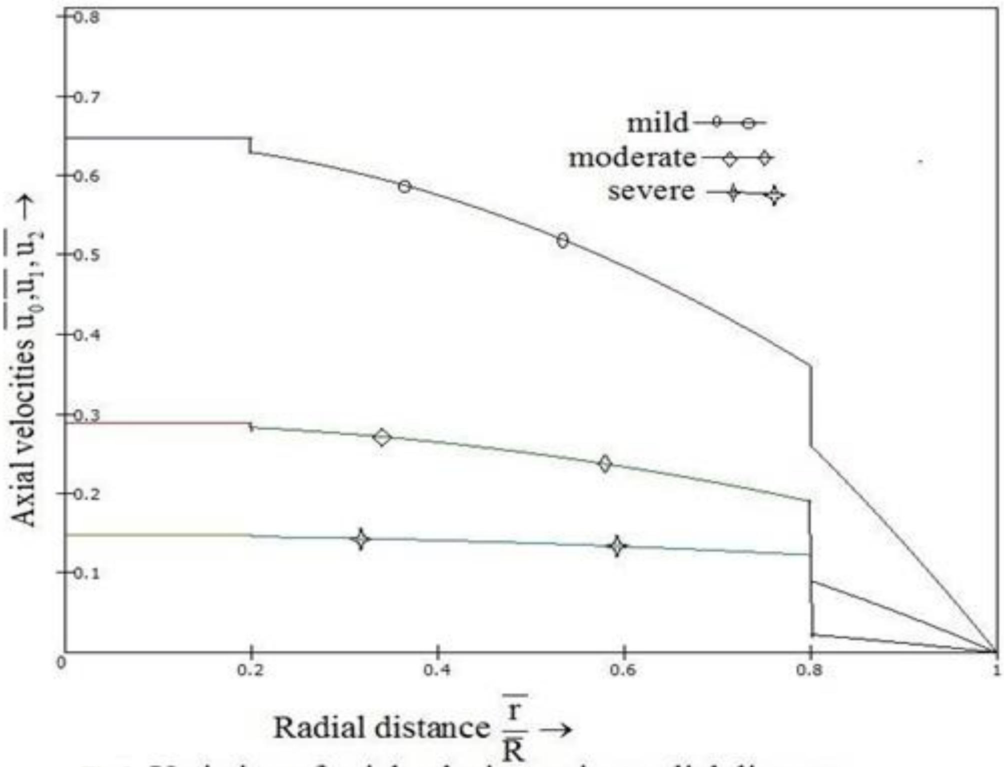

Fig.2: Variation of axial velocity against radial distance for $u_s = .1, \tau_0 = 0, n = 2$

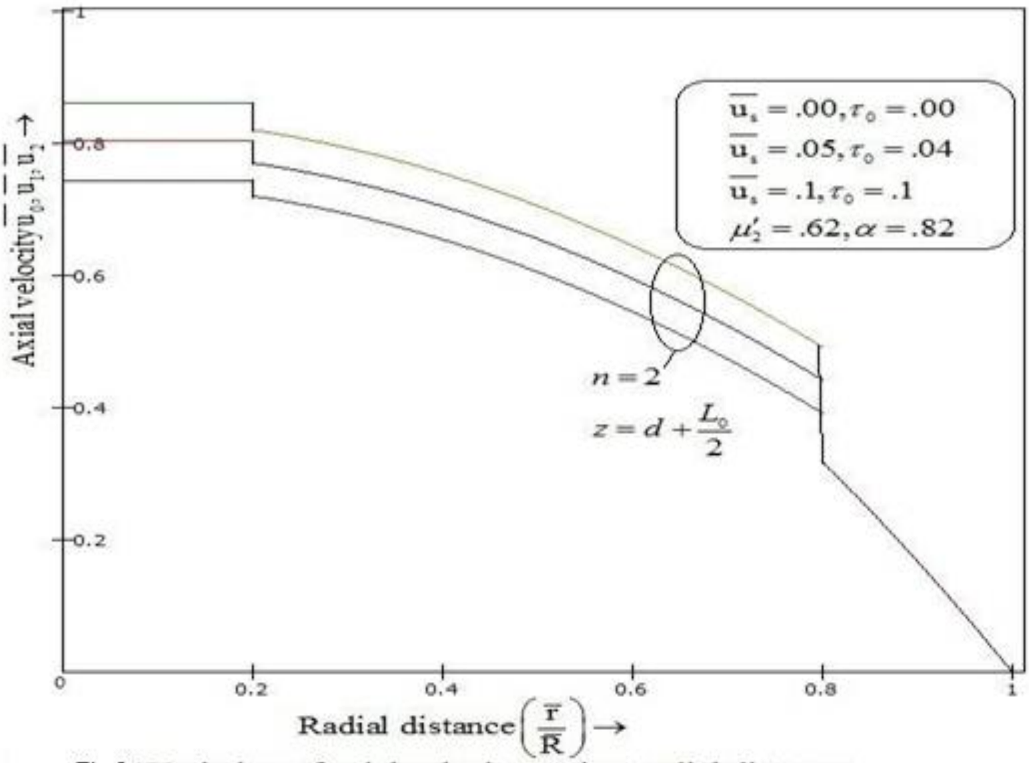

Fig.2 : Variation of axial velocity against radial distance for different slip velocities and n=2

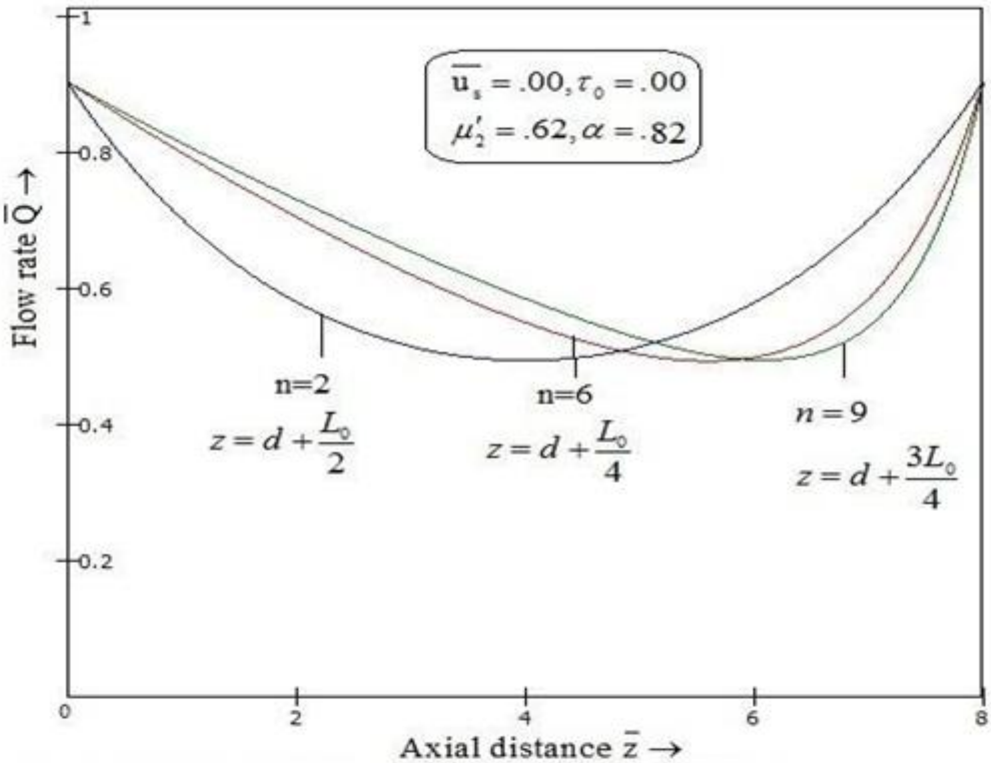

Fig.4 : Variation of flow rate against axial distance for different values of n

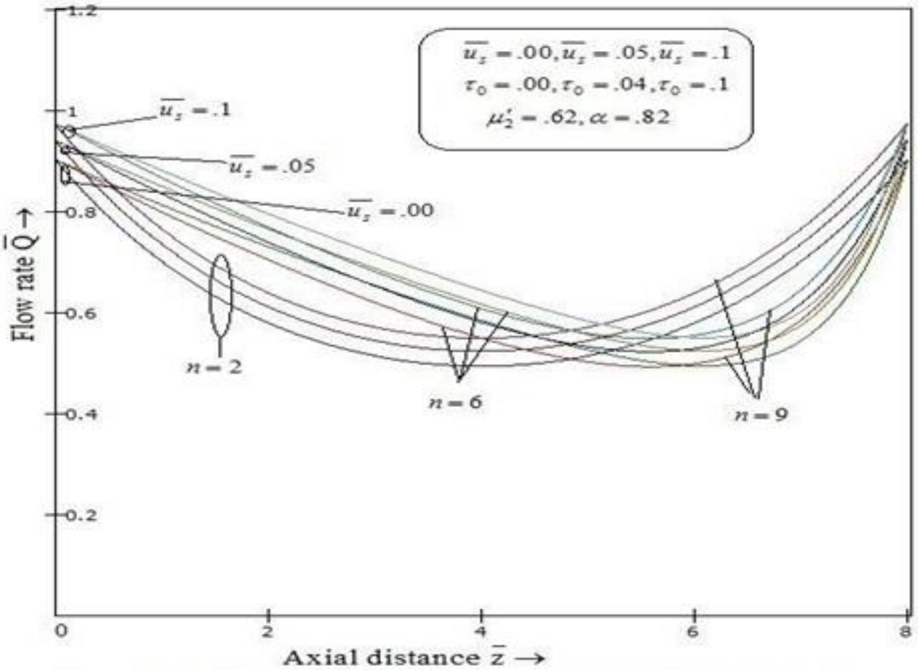

Fig.5 : Variation of flow rate against axial distance for n =2,6,9 and $\overline{u_s} = 0, .1, .05$

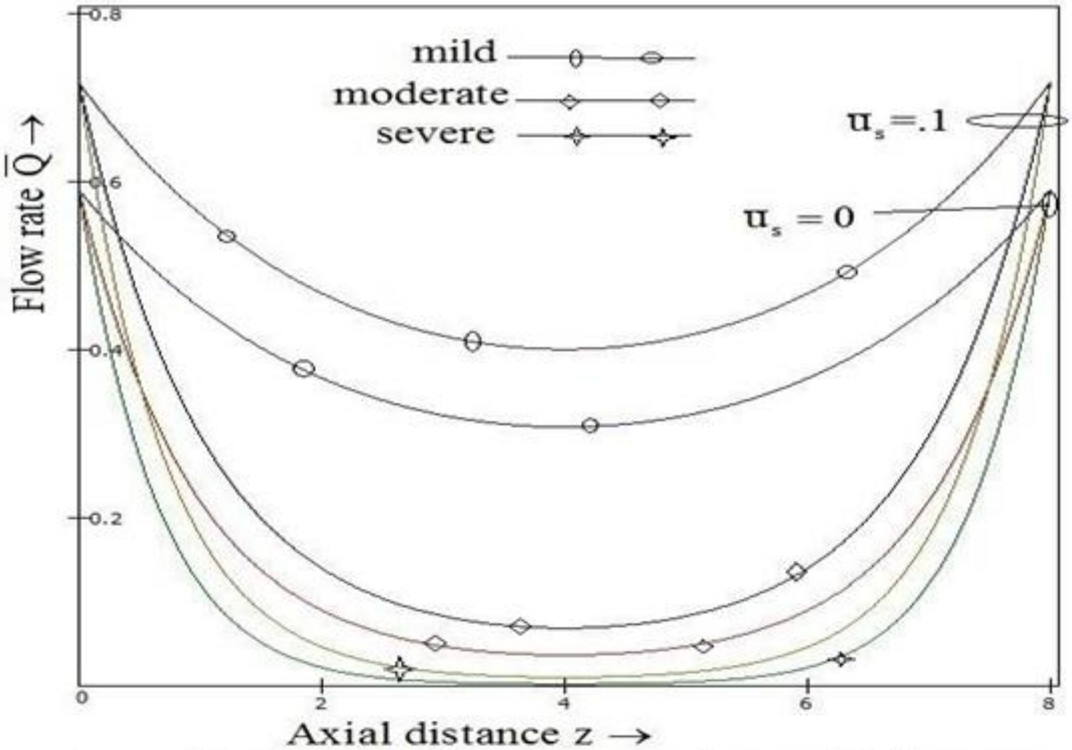

Fig.6 : Variation of flow rate against axial distance for $\overline{u_s} = 0,.1, n = 2$

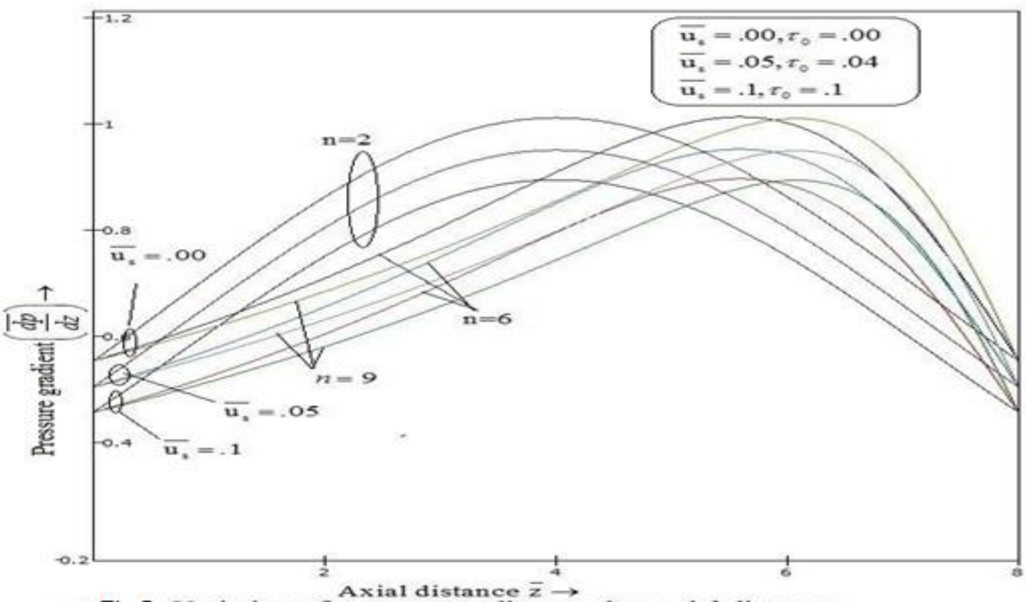

Fig.7 : Variation of pressure gradient against axial distance for different values of n and $\overline{u_s} = .00,.05,.1$

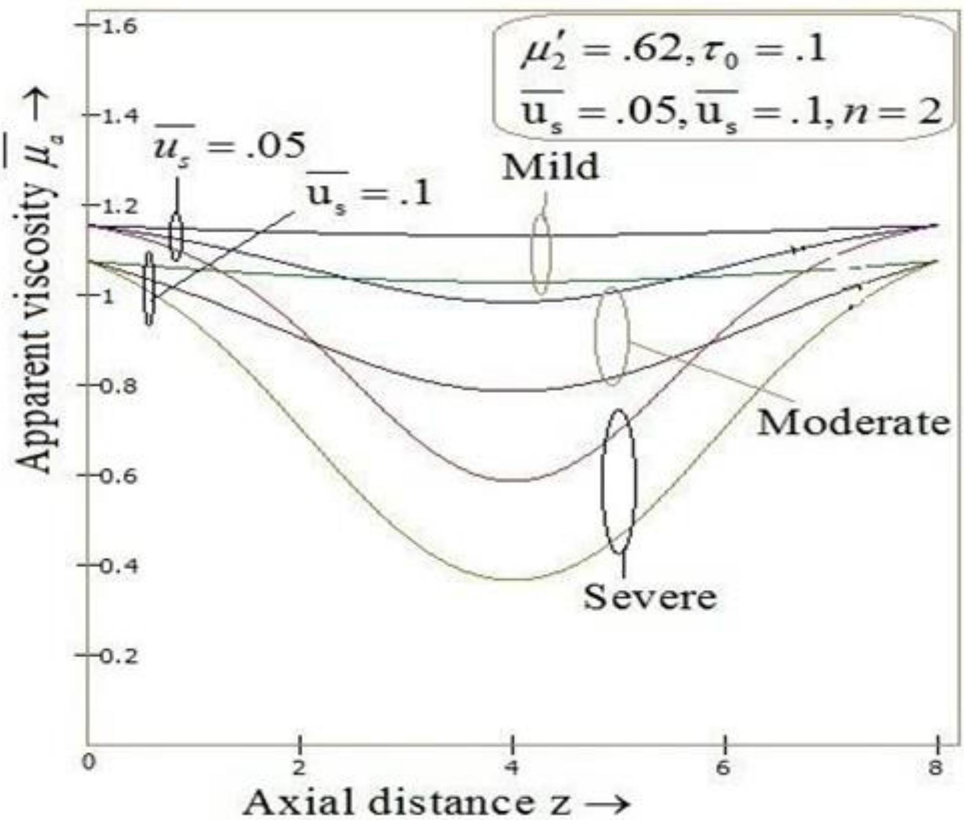

Fig.8: Variation of apparent viscosity against axial distance for different slip velocities

Results and Discussions

The core of this two-layered investigation for steady blood flow within an asymmetric stenosed artery is assumed to be represented by Bingham fluid with viscosity =2cp, and the peripheral layer is accounted for by Newtonian fluid, having a shear viscosity 1=1.2cp (Fig.1). A slip condition at the interface of the two fluids and the typical zero-slip at the stenotic vessel wall are employed in the model. In cases of mild, moderate, and severe growth of an arterial stenosis, an axial slip for velocity has been applied at the interface of fluids. A finite yield stress is inhabited by the Bingham fluid, which is a mixture of visco-inelastic, non-Newtonian fluids. Such a fluid is said to be impermeable if the shear stress rz at a given yield stress is below a particular value, and proportionate to the increase in shear rate if it exceeds the yield value 0 (> 0). Due to their innate fluid qualities, there may be two separate scenarios in which their flow behaviour occurs.

(A) If $\tau_{rz} < \tau_0$ that is, if shear stress at a critical distance r is not more than its yield value—blood won't flow inside the circular s ystem.

(b) Blood flow inside the body is conceivable if rz > 0 (i.e., shear stress is not lower than a finite yield stress). There may therefore be three regions for flow along the confined uniform zone of an artery in accounting or the Bingham behaviour of body fluid blood in two-layered flow, namely, 0 r r0, r0 r R1(z), and R1(z) r R(z). It is possible to derive analytical formulas for velocity, flow rate, pressure gradient, flow resistance, wall shear stresses, and apparent viscosity. It is evident that velocity (in equations 7.2.26-7.2.28) depends on shear viscosities 1,, radial r and axial z coordinates, tube radii R0, R(z), and R1(z), pressure gradient c, stenosis dimensions, and velocity slip u. If R1(z) = R(z), then 0 = 0 and 1 = 1.

A one-layered Bingham model of an arterial stenosis with slip or no slip at the vessel wall is produced by the current model. It reduces to the Poiseuille flow model with slip and zero-slip at the tube wall for R(z)= R0 =R1(z) and 0 =0 (Schlichting, 1968). It leads to stenosed Newtonian fluid models for one-layered blood flow with slip or zero-slip when 0 = 0 and R1(z)=R(z), and to a Newtonian model when 0 = 0 and R1(z)=R(z).