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Application of Chaos and Nonlinear Dynamics in Secured Optical Communication Systems

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Abstract

In certain circumstances, chaotic oscillations can be seen in a nonlinear system with feedback. The behavior of chaotic systems is uncertain due to their extremely complicated evolution throughout time. Chaotic paths separate from one another over time. As such, safe cryptography is where chaos theory is most commonly applied. Several optical devices that have been investigated in literature for producing optical chaos include lasers, Mach Zehnder interferometers, and acoustic-optical systems incorporating electronic feedback. Studies have shown that feedback in nonlinear systems can be a useful tool for characterizing chaotic paths. Measures that express the degree of uncertainty in chaotic dynamics, such as Kolmogorov entropy or Lyapunov Exponents, are frequently used by researchers to quantify chaos. Every dynamic system's trajectory changes when noise is present, thus in order to understand noise's impacts in this context, I read through a number of research publications, as shown in the section below. With the use of nonlinear optical devices, this work intends to demonstrate the uses of chaos in a chaos-based communication strategy and to successfully retrieve the message from the receiver side. Ultimately, a gap analysis is carried out to identify the gaps in current research trends and to suggest directions for further investigation.

Keywords: Chaos, Optics, Entropy, Bifurcation, Communication

1. Introduction

Optical communication networks are popular for THz bandwidth and high consistency universally. With the explosive evolution of optical network resources, secure optical communication is becoming an important issue. Various methods for safe optical communication have been proposed over the past few years[1][2]. The advantages of optical communication are high bandwidth, remarkably low transmission loss, great transmission range, and no electromagnetic interference. Among them, chaotic communication is considered a promising technique for secure optical communication[3].Since1970, optical wired communications have been developed as one of the most popular methods of communication. Optical communication systems comprise the following parts, (i)Transmitter (ii)Receiver, and (iii)Optical Fiber as a channel. Fiber optics has enormous applications in the fields of military and aerospace applications. These are also used in hydrophones for SONARs, and seismic applications [4]. Optical wireless communication (OWC) is evolving with enormous potential for the formation of consistent communications networks. OWC has three parts namely, (i) Transmitter (ii) Receiver (iii) Channel. The transmitter produces the carrier signal inside which the message signal is modulated. The sensed electrical signal at the receiver side is handled for extraction of the actual communicated message. Vast applications of OWC are found in the areas of the interior, vehicular, worldly, submerged, intersatellite, deep space, etc. OWC has exclusive advantages to offer safe, protected, low-priced, and high-bandwidth communications. Keeping this prospect of optical communication in mind I have done a rigorous literature review on application of chaos theory in Optical communication system.

2. Literature Review

Chaos is a fundamental aspect of a nonlinear dynamical system with feedback. Chaos is highly sensitive to initial conditions. A small change in the initial condition results in a huge amount of divergence in chaotic trajectories[1]. Chaos is found in complex systems such as weather, astronomy, politics, and economics. Chaos seems to behave randomly. These underlying characteristics of chaos are extensively utilized in spread spectrum communications and several cryptographic schemes. This chapter of the thesis introduces the elementary ideas of nonlinear dynamics and gives an outline of research in the field of communication with chaos[2][3]. Chaotic optical communication plays a vital role in our daily life[4].

In [5], the authors define a different model for optical communication systems along with different simulation tools. In the chaotic cryptography system, one of the sources is used for the transmission, i.e., to conceal the message and the other one is used at the receiver for message recovery. Doing proper synchronization between transmitter and receiver is the key for recovery of the message signal on the receiver side[6]. Synchronization can be attained by optically injecting a fraction of the emitter laser output into the receiver laser, which, under appropriate conditions, regenerates the chaotic signal of the transmitter. Extraction of information is simply accomplished by the difference between the signal coming from the emitter and the chaotic signal reproduced at the receiver[7]. The whole concept is explained in Figure 2.1. However, it is very challenging, for a decipher, to retrieve the message because perfect synchronization depends on the use of two lasers with the same parameters[8].

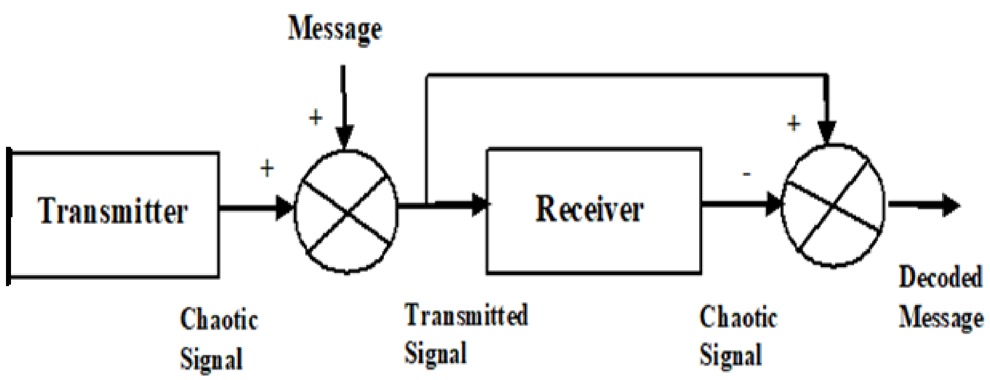

Figure-1 Schematic diagram of chaos masking technique.

Wideband information carrier signals with a high degree of robustness and confidentiality have been identified as chaotic signals. Systems that rely on chaos experience extremely little electromagnetic interference. Another method of encoding information that has been proposed is the use of chaotic signals. Pecora and Carroll suggested the idea, and Cuomo and Oppenheim electronically accomplished it in 1993[7]. Following these articles, there is now a chance to use these strategies to improve communication privacy[6][10].

The first use of chaos in optical communication systems was proposed in [11] for coherent optical communications. Coherent optical communication schemes have better efficiency than non-coherent systems theoretically, but the condition is that receiver can genuinely replicate the transmitter side chaos. Synchronization is the process by which this replication is done[11]. Digital chaos shift keying (DCSK)[12] system is the most commonly considered non-coherent chaos-based digital communication scheme. An overview of different types of digital communication systems, employing nonlinear dynamics has been explained by the authors in [12]. using chaos and nonlinear dynamics. Both OWC and fiber optical communications techniques are discussed in that work. Different types of chaos-shifted-keying techniques are also illustrated in these two books [13][14].

In [14,15] laboratory-based demonstration of chaos has been done. It has proved its potential in standard fiber or optical wireless based non-coherent Optical communication systems also. It provides us with the simplest ways of generating very high-dimensional chaotic carriers that offer a very high level of safety and also good transmission rates of information. In [15], the methods for the generation of chaos are described and in [16], their use in chaos encrypted optical communication links. AO modulator-based chaos has also been deployed to realize a chaos-based optical communication link[17]. Different aspects of such a communication setup have been taken into consideration in the references[18] and [19].

Detecting the existence of chaos in a nonlinear system is a significant problem that is resolved by quantifying the LLE[20]. LE computes the exponential deviation of initially adjacent state-space trajectories and approximates the quantity of chaos in a system. In [21] the author has examined the fundamentals of chaos and chaos-based dynamics. For characterizing a chaotic system, computation of LE is essential[22]. The reference[21] gives an idea about the computational techniques for LE for different types of nonlinear map. When the mathematical equations defining the dynamical system do not exist, one can estimate the entire LE from an investigational time series[23][24]. References [23] and [24] present a new way of computing the LE from an investigational time series. The method uses the definition of the LLE and it is precise because both of them take benefit of all the obtainable data. The process is fast and also easy to implement.

In [25] Ghosh and Verma have shown how to deduce the expression of LE of an AO modulator with electronic feedback to portray the characteristics of optically generated chaos w.r.t to the system parameters. Later on, the author has also demonstrated the chaotic signals as a random process with a beta distribution and shown that their characteristics or statistical properties alter with time.

Bifurcation analysis is an influential technique for studying the steady-state nonlinear dynamics of systems. A bifurcation diagram gives us visual information regarding the succession of period-doublings i.e. periods 2,4,8 etc, produced in cascade order with the change of system parameters[26][27]. A bifurcation accounts for the qualitative change in system dynamics with the change of system parameters. References [26] & [27] have explored the bifurcation phenomenon for analysing the chaotic dynamics of a nonlinear system. Different strategies related to bifurcation have been investigated in these papers via rigorous numerical experiments[28].

Optical wireless communications (OWC) have attained great attention over the last few decades. In some cases, OWC is understood as a substitute for next-generation technologies. Therefore, OWC is being extensively installed in numerous indoor, outdoor, interplanetary communication, and submerged systems. Submerged wireless information transfer is of extreme interest to the armed, engineering, and technical community, as it plays a vital role in strategic surveillance, pollution monitoring, oil rigs and their maintenance, offshore examinations, temperature variation monitoring, and oceanography study[29].

Various kinds of nonlinear optical systems have been explored to create chaotic optical waveforms as discussed in an earlier section. EO or AO modulator-based systems possess nonlinear sinusoidal characteristic[17,19,30,31]. Here in this thesis we have designed, fabricated, and tested an analog computing circuit mimicking the characteristics of an EO Modulator for the generation of chaos and finally, a secure chaos-based communication link has been built up by deploying such an analog electronic simulator both as transmitter and receiver.

3. Chaos in Communication

Broadband signals with terahertz bandwidth and low power density are known as chaotic communication signals. The samples that are directed through the channel in chaotic communication systems are components of chaotic signals. Chaotic optical communication has many qualities that make it useful for communication, including its nonlinear and aperiodic behavior. Communication signals based on chaos are difficult to decipher and intercept[36].

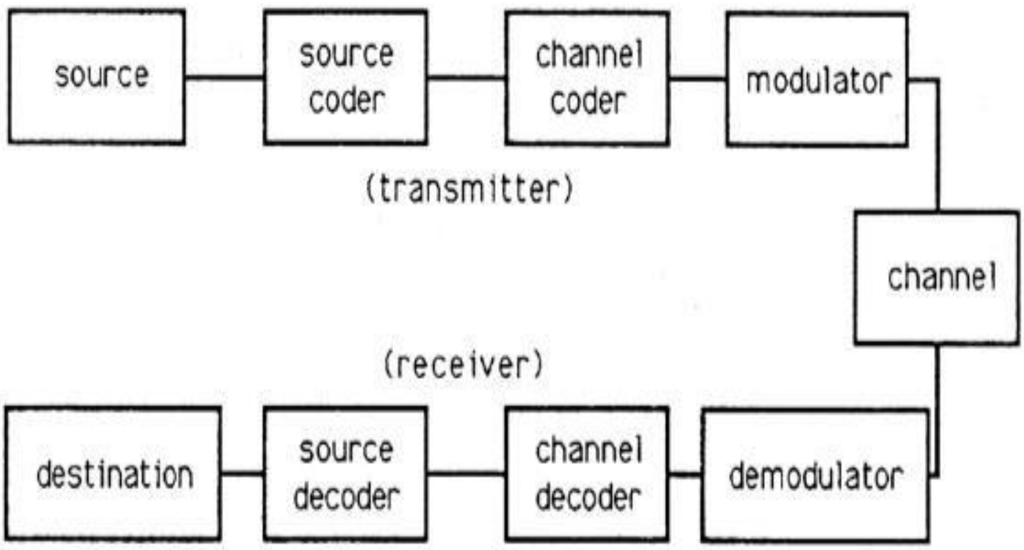

Figure.2 Digital chaotic communication system.

4. Quantification of Chaotic Complexity using Entropy

To create chaotic optical signals, a vast array of nonlinear optical devices is used. These include systems with electronic feedback, such as (a) lasers, (b) electro-optical (EO), and (d) acousto-optical (AO) systems. Therefore, for secure communication on nonlinear optical devices like EO or AO systems, a chaos-based communication technique can be easily implemented. It alters the information signal so that only a designated recipient who is aware of the transformation settings beforehand is able to access the data. In a chaos masking setup, the signal is embedded inside a chaotic carrier that is created at the transmitter in order to conceal the information. Message extraction is done on the idea of proper synchronization between transmitter and receiver. After the synchronization task is carried out, the decryption of the information is carried out by deducting the regenerated chaotic signal from the transmitted signal. The whole scheme is explained in Figure-2. Privacy in chaotic communication systems is maintained by selecting identical chaotic parameters both for transmitter and receiver.

Lyapunov Exponent(LE)is being used to define Electro Optically or AcoustoOptically generated chaos. For quantifying the amount of 'uncertainty' in the chaotic waveform we have taken the help of LE. It is known that a nonlinear system generates chaotic oscillations for the positive value of LE and stable periodic output otherwise[37]. For example, I have considered the case of an EO modulator of Figure-3 represented by equation-1with two different sets of system parameters i.e. the bias factor *a*, net feedback gain *b*, the intensity of the input light beam I_{in}, and initial condition *Y(0)*.

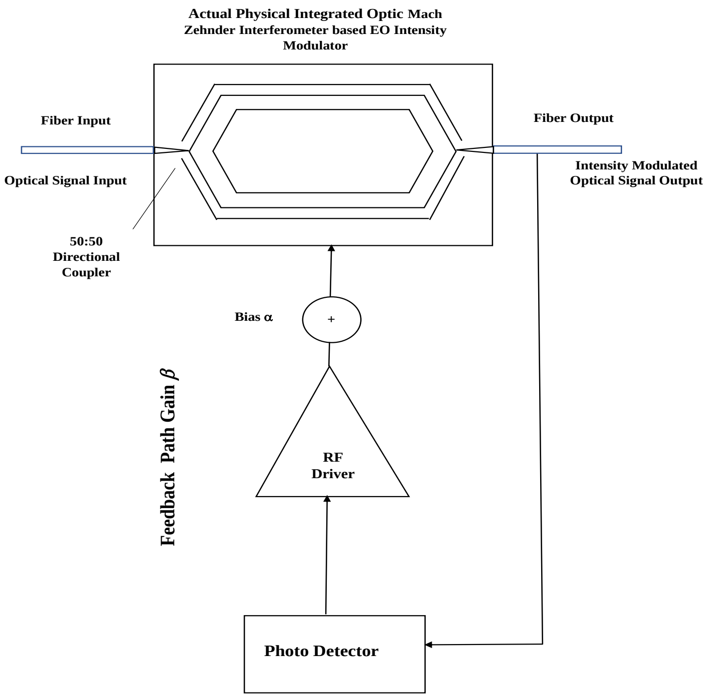

Figure.3.A schematic diagram of Electro Optic system with feedback[]

$$Y(n) = I_{in}\cos^2[\alpha + \beta Y(n-1)] \quad (1)$$

Values of the parameters are mentioned in Figures.4 (a) and (b). Parameters are chosen arbitrarily. For the first set of system parameters output *Y(n)* converges to a steady-state value as the value of LE is found negative. On the contrary, output under goes chaotic oscillation for the positive value of LE as shown in Figures.4 (a) and (b). Steps for calculation of LE for a nonlinear recursive system are given below

$Y(\mathrm{n}) = f[Y(\mathrm{n}-1)]$ (2)

Then the trajectory of the map of Equation (1.1) is given by

$Y(n) = f^n[Y(0)] = f[f[...f[Y(0)]...]]$ (3)

When the initial condition is perturbed to $Y'(0) = Y(0) + \mathrm{e}$, for small e, the sequence generated by Eq. (1.2) is given by

$Y'(n) = f^n[Y(0)+\varepsilon]$ (4)

The Lyapunov Exponent λ of a nonlinear one-dimensional map in Equation. (1.1) is defined as follows

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln \left| \frac{df[y]}{dy} \right|_{y=Y(k)} \quad (5)$$

Hence the expression for LE of EO map of equation (1) can be written as

$$\lambda = \ln|\beta I_{in}| + \lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln|\sin[2\alpha + 2\beta Y(k)]| \quad (6)$$

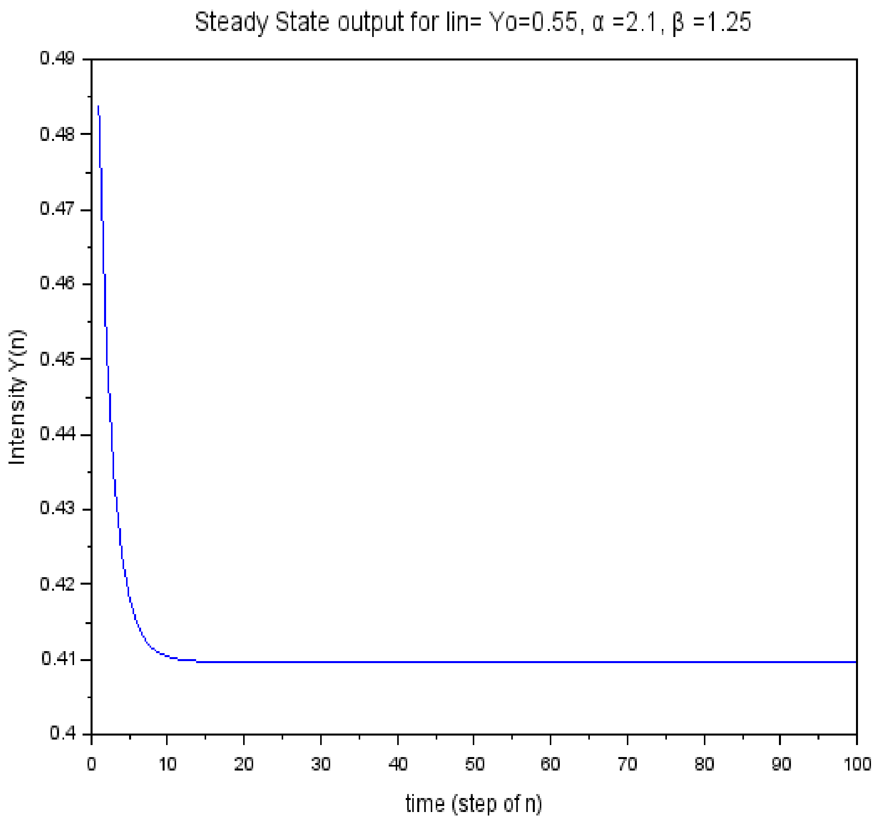

Figure-4 (a) Y(n) Converges to a steady state value when LE<0(LE=-0.8)

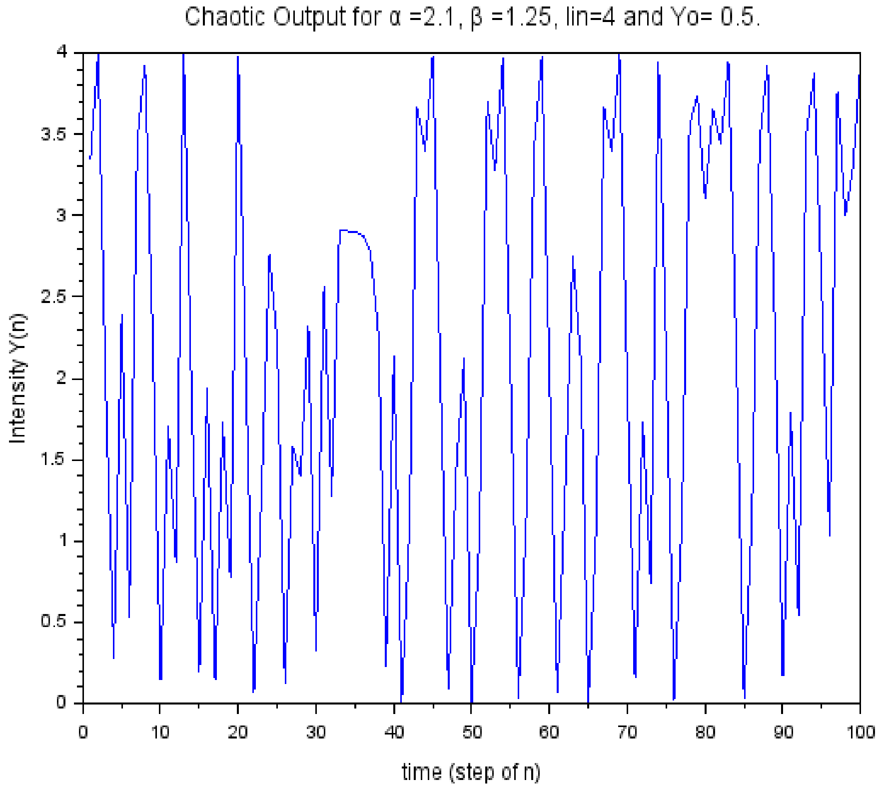

Figure-4 (b) Y(n) undergoes chaotic oscillation when LE>0(LE=0.51)

5. Characterization of Chaos using Bifurcation Analysis Technique.

The bifurcation analysis technique can also be used to characterize the chaos that arises from any nonlinear optical system, such as an EO or AO system[9][10]. When system parameters are altered, the dynamic system's qualitative structure also shifts. Bifurcations are these qualitative shifts in the dynamics. For instance, the bifurcation diagram of the EO system with regard to parameter *b* is shown in Figure 5. When system parameters are altered, the dynamic system's qualitative structure also shifts. Bifurcations are the places where there are qualitative shifts. The bifurcation diagram makes it simple to identify the bifurcation points. It may be quickly noticed the values of *b* at the bifurcation points by zooming in on the Figure and adjusting the scale along the x-axis. One can identify periods 1, 2, 4, 8, and so on by magnifying the x-axis. In this manner, periodic points rise with the increased value of *b*. At *b* = 3.7313 infinite oscillations are found making the system chaotic, Figure-6 with positive LE ensures the fact.

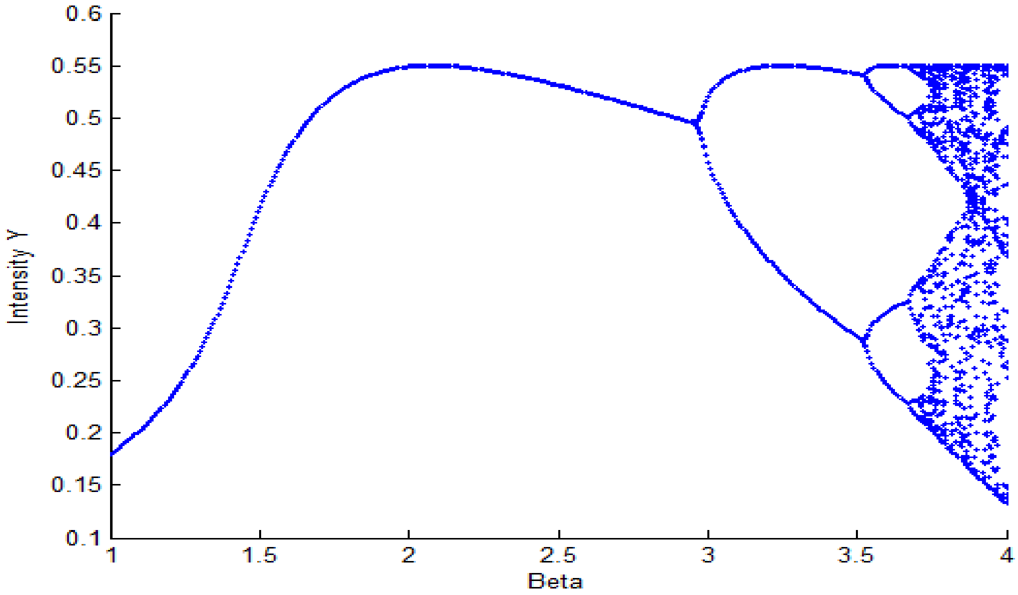

Figure-5. Bifurcation diagram of EO system w.r.t parameter*b*

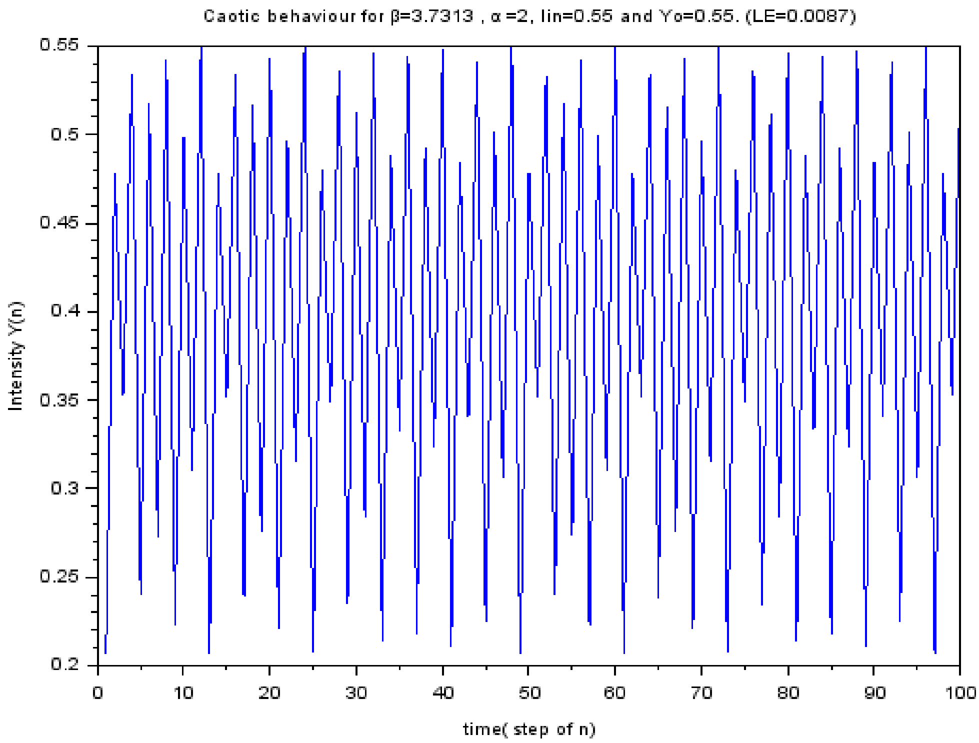

Figure-6 System shows the chaotic behaviors for β=3.7313 with α =2, I_{in}=0.55 and Yo=0.55. (LE=0.0087)

6. Noise Immunity of EO Map

Study of noise in the dynamical system is a significant area of research. Noise can remarkably alter the dynamics of chaotic systems[11]. In [12] we have investigated here the noise immunity property of the EO map by imposing thermal noise into the system and consequently increasing the intensity of the noise. After the examination, we have found out that the EO system is extremely immune to noise. This noise immunity property of EO modulator is quantified with the help of mathematical derivations in that piece of work[12].

In the above mathematical analysis, we have found out that this immunity to noise arises from the sinusoidal $\cos^2[x]$ nature of nonlinearity present in the right-hand side of the chaotic map of equation (1).

Table-1 Variance of noise signal vs variance of EO map

Variance of noise signal (□ □ □	Variance of noisy EO map $\mathrm{Var}\{\cos^2[\eta(n)]\}$
0.01	$1.9\text{x}10^{-4}$
0.0001	$1.99\text{x}10^{-8}$
1	0.1250
100	0.1250
10000	0.1250
1000000	0.1250

An electrical signal's random fluctuations are known as noise, and they are a feature of all electronic circuits. Electronic device noise varies widely since it comes from a variety of sources. When the temperature is not zero, thermal noise is inevitable. Errors or unwanted random disruptions of a valuable information signal are referred to as noise in communication systems. Noise frequently affects communication systems. Noise in communication systems reduces the quality of the received signal; however, we have demonstrated that this EO system may be used to establish an effective communication link because of its high noise tolerance.

7. Research Gap Analysis

To develop a chaotic communication link, it is essential to quantify the complexity of chaos either generated electronically or optically. Hence to characterize different types of nonlinear optical systems like AO and EO systems etc, authors of different write up have taken the help of entropy called LE. Along with the characterization of a nonlinear system, we have seen that by varying system parameters like bias factor, feedback gain, input intensity, and initial condition exciting the AO or EO cell one can change the value of LE and ultimately able to control the system dynamics. Instead of using LE, we may also take the help of topological entropy (TE) to detect the presence of chaos in a nonlinear system. TE measures the dynamical complexity as the system evolves with time. It is common practice to characterize a chaotic system using LE. The same chaotic system may be measured with the help of TE and ultimately comparison may be done between these two values of entropies.

It has been shown in study, how to estimate entropy called LE from investigational time series for a mathematically undefined system. Rosenstein Algorithm has been explored efficiently for the said purpose. Operational steps may be counted for this algorithm when applied to the selected map i.e. EO

or AO system. These $\cos^2$ or $\sin^2$ maps are bounded in nature. In this connection, a relative study may be done for a different kind of map, concerning the number of operational steps. Apart from this algorithm, other popular algorithms like the Grassberger-Procaccia Algorithm, Wolf's Algorithm, Sato's Algorithm, etc. may also be explored for calculation of LE from experimental data. A comparative study may be carried out among the values of LE computed with the help of these different algorithms
Throughout this survey, it has been found that optical chaotic signals are analyzed in the time domain. DFT or FFT operation may be done on the signal generated from a nonlinear optical system and some numerical experiments may be conducted to demonstrate the frequency domain characteristics of a chaotic signal. LE or TE may be calculated for the signals converted into the frequency domain.
A piece of information could be encrypted and decrypted inside a broadband chaotic carrier using a couple of nonlinear optical devices, messages are implanted inside a chaotic carrier in the transmitter and extracted at the receiver. This ideology has been employed in different literatures to realize an experimental setup for the chaos masking communication scheme. In this regard, hardware circuits may be developed based on different types of Chaos Shift Keying schemes to study the performance of such communication networks realized with the help of electronic simulators as mentioned in earlier section.

8. Conclusion:

In this study, a chaos masking communication technique has been developed using the previously mentioned electrical simulators for EO modulators as both transmitter and receiver, taking into consideration the possibility for chaotic signals to conceal information. In communication networks, chaotic optical communication enhances security and privacy. At the hardware level, it synchronizes chaotic optical transmitter and receiver devices in order to encrypt and decrypt communications. This new technology can be combined with nonlinear optical devices to provide higher levels of security than can be achieved with standard cryptography techniques.

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