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Thermal Radiation on Hydro Magnetic Visco-Elastic Fluid Flow with Variable Thermal Conductivity over Non-Isothermal Stretching Sheet

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Abstract

In the present paper we contemplate to study the effect of variable thermal conductivity on thermal radiation of a MHD flow of a visco-elastic fluid over a stretching sheet. In contrast to the work of MHD Visco-Elastic Fluid Flow and Heat Transfer Over a Non- Isothermal Stretching Sheet With Radiation the present work considers the effect of variable thermal conductivity and internal heat generation and absorption.

Keywords :Variable Thermal Conductivity, MHD Flow, Visco-Elastic Fluid, Stretching Sheet, Thermal Radiation, Heat Transfer, Internal Heat Generation, Internal Heat Absorption, Magnetohydrodynamics, Non-Isothermal Stretching Sheet, Viscoelastic Fluid Flow, Thermal Conductivity Variation, Heat Transfer Enhancement, Radiation Effects, Fluid Flow Modeling, Heat Absorption and Generation, Magnetic Field Effects, Thermal Boundary Layer, Heat Flux, Thermal Gradient.

Introduction

In recent years, the study of Magneto hydrodynamic flow and heat transfer problems has gained considerable interest because of its extensive engineering applications. To be more specific, it may be pointed out that, many industrial processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. During the process these strips are sometimes stretched. Mention may be made of annealing and tinning of copper wires. In all these cases, the properties of final product depend largely on the rate of cooling. By drawing such strips in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and final product of required characteristics can be obtained. Another important application of hydro-magnetic to metallurgy lies in the purification of molten metals from non-metallic inclusion by the application of magnetic field.

Motion of Newtonian/non-Newtonian fluids in the presence of transverse magneticfield was studied earlier by Sarpakaya (1961), Djukic (1973, 1974), Pavlov (1974), Chakrabarti and Gupta (1979), Soundalgekar and Takhar (1980), investigated the effects of uniform transverse magnetic field on forced and free convention flow past a semi-infinite plate taking into account of viscous dissipation and stress work. Raptis and Tzivnides (1983) carried out analytical investigations on free convective flow pastan infinite vertical surface when the fluid is electrically conducting in the presence of an external transverse magnetic field. Mahesh Kumari et al. (1990) studied the flow and heat transfer over a stretching sheet in an electrically conducting fluid, which is at rest. The magnetic field is applied parallel to the sheet.

They included the effect of the analysis of induced magnetic field and sources or sinks. Vajravelu and Nayfeh (1992) studied the hydro magnetic flow of a dusty fluid over a stretching sheet including the effects of suction. Vajravelu and Rollins (1992) examined the heat transfer characteristics in an electrically conducting fluid over a stretching sheet with variable temperature and internal heat generation or absorption. Andersson (1995) presented an exact similarity solution for velocity and pressure of the two-dimensional Navier-Stokes equations. Samlawrence and Nageswara Rao (1996) studied a steady incompressible two-dimensional MHD boundary layer flow near a symmetric plane stagnation point. Chiam (1995) analyzed the boundary layer flow due to a plate stretching with a power-law velocity distributions in the presence of transverse magnetic field Chiam (1997) presented the solutions of the energy equation for the boundary layer flow of an electrically conducting fluid under the influence of transverse magnetic field over a linearly stretching non-isothermal flat sheet.

However, all these researchers restrict their analysis to hydro magnetic flow and heat transfer. None of them deals with the much more intricate problem involving the effect of thermal radiation on a hydro magnetic visco-elastic fluid flow. Available literature on thermal radiation, Raptis and Perdikis (1998), Perdikis and Raptis(1996), Raptis (1999) and Chamkha (2000) shows that the work is not carried out for visco-elastic fluid of the type Walter's liquid B where the thermal conductivity is a function of temperature. Keeping this in view, in the present chapter we contemplate to study the effect of variable thermal conductivity on thermal radiation of a MHD.

Now of a visco-clastic fluid over a stretching sheet. In contrast to the work of chapter four the present work considers the effect of variable thermal conductivity and internal heat generation and absorption.

Mathematical Formulation

Consider a steady, laminar flow of an incompressible and electrically conducting visco-elastic fluid over a semi-infinite, impermeable stretching sheet. Two equal and opposite forces are introduced along the x-axis so that the sheet is stretched with a speed proportional to the distance from the origin. The resulting motion of the otherwise quiescent fluid is thus caused solely by the moving surface. A uniform magnetic field of strength B_0 is imposed along y-axis. This flow satisfies the rheological equation of state derived by Beard and Walters in 1964. The steady two-dimensional boundary layer equations for this flow in usual notation are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} \qquad (2)$$

In deriving these equations, it is assumed, in addition to the usual boundary layer approximations, that the contribution due to the normal stress is of the same order of magnitude as the shear stress. The boundary conditions applicable to the flow problem are,

$$u = bx \;\; v = 0 \; at \;\; y = 0$$
$$u \to 0, u_y \to 0 \; as \; y \to \infty (3)$$

Here x and y are respectively the directions along and perpendicular to the sheet, u and v are the velocity components along x and y directions. $\rho, \gamma, B_0, \sigma and k_0$ are respectively, the density, kinematics viscosity, coefficient of visco-elasticity. Solely the stretching of the sheet, when the free stream velocity is being zero causes the flow. Equations (1) and (2) admit a self-similar solution of the form

$$u = bxf_\eta(\eta), \; v = -\sqrt{b\gamma} f(\eta), \quad \eta = \sqrt{\frac{b}{\gamma}} y (4)$$

where subscript η denotes the derivative with respect to η. Clearly u and v satisfy equation (1) identically. Substituting these new variables in Equation (4), we have

$$f_\eta^2 - f f_{\eta\eta} = f_{\eta\eta\eta} - Mn f_\eta - k_1 \{2 f_\eta f_{\eta\eta\eta} - f f_{\eta\eta\eta\eta} - f_{\eta\eta}^2\} (5)$$

where $k_1 = \frac{k_0 b}{\gamma}$, is the visco-elastic parameter and $Mn = \frac{\sigma B_0^2}{\rho b}$ is the magnetic parameter.

The boundary conditions (3) become

$$f_\eta = 1, f = 0 \quad at \;\; \eta = 0$$
$$f_\eta \to 0, f_{\eta\eta} \to 0 \;\; as \; \eta \to \infty \; (6)$$

An exact solution of equation (5) satisfying the boundary conditions (6) is given by

$$f(\eta) = \frac{1 - e^{-\alpha\eta}}{\alpha}, \qquad \text{With} \quad \alpha = \sqrt{\frac{1 + Mn}{1 - k_1}}. \qquad (7)$$

Therefore, the velocity components are

$$u = bxe^{-\alpha\eta}, \;\; v = -\sqrt{b\gamma} \frac{1 - e^{-\alpha\eta}}{\alpha} \qquad (8)$$

Heat Transfer Analysis

The energy equation in the presence of radiation and internal heat generationabsorption for two-dimensional flow is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{q}{\rho c_p}(T - T_\infty) - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} (9)$$

Where $T, \rho, q and c_p$are, respectively, the temperature, the density, and the volumetric rate of heat generation and specific heat at constant pressure; k, the thermal conductivity is assumed to vary linearly with temperature. We assume that thermal conductivity k is of the form

$k = k_\infty(1+\varepsilon\theta(\eta)), where \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$ (PST case) (10)

k= $k_\infty(1+\varepsilon g(\eta)), where g(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$ (PHF Case)

and $\varepsilon = \frac{k_w-k_\infty}{k_\infty}$ is a small parameter. By using Roseland approximation [Brewster in 1992] the radiative heat flux is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \quad (11)$$

Where $\sigma^* and k^*$are, respectively, the Stephan –Boltzmann constant and the mean absorption co-efficient. We assume that the differences within the flow are such that T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (12)$$

Now using Equation. (10), (11) and (12), Equation. (9) becomes,

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{q}{\rho c_p}(T-T_\infty) - \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2} \quad (13)$$

The thermal boundary conditions depend upon the type of the heating process being considered. Here, we are considering two general cases of non-isothermal boundary conditions namely, (1) Prescribed surface temperature (PST) and (2) Prescribed wall heat flux (PHF), varying with the distance.

Prescribed Surface Temperature (PST Case)

For this heating process, the boundary conditions are ,

$$T = T_w = A\left(\frac{x}{l}\right)^r \quad at \quad y = 0$$

$$T = T_\infty \qquad as \quad y \to \infty \qquad (14)$$

where l is the characteristic length, A is a constant and r is the wall temperature parameter. Using (8), (9) and (14), the dimensionless temperature variable θ given by (10), satisfies

$$\boldsymbol{(1+\varepsilon\theta+Nr)\theta_{\eta\eta} + Pr\, f\, \theta_\eta - Pr(rf_\eta - \beta)\theta + \varepsilon\theta_\eta^2} = \mathbf{0} \quad (15)$$

Where$Pr = \frac{\mu c_p}{k_\infty}$is the Prandtl number $Nr = \frac{16\sigma^* T_\infty^3}{3k^* k_\infty}$is the radiation parameter and$\beta = \frac{q}{\rho c_p b}$ is the heat source / sink parameter. The corresponding boundary conditions are

$$\theta(0) = 1, \quad \theta(\infty) = 0. \quad (16)$$

Prescribed Heat Flux (PHF Case)

In this case the boundary conditions are

$-k\left(\frac{\partial T}{\partial y}\right) = q_w = D\left(\frac{x}{l}\right)^s$ at y = 0, (17)

T → T∞ as y → ∞

Here D is a constant and s is a wall heat flux parameter. Defining

$g(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$, where $T_w - T_\infty = \frac{D}{k}\left(\frac{x}{l}\right)^s \sqrt{\frac{\gamma}{b}}$, (18)

and using (7.2.8), (7.2.18) and (7.2.10), (7.2.9) becomes,

$(1+\varepsilon g + Nr)g_{\eta\eta} + Pr\, f\, g_\eta - Pr(sf_\eta - \beta)g + \varepsilon {g_\eta}^2 = 0$ (19)

and the boundary conditions become

$g_\eta(0) = -1, g(\infty) = 0$ (20)

Perturbation Analysis

We follow a perturbation expansion approach to solve equation (7.2.15) (PST Case). Suppose

$\theta = \theta_0(\eta) + \varepsilon\theta_1(\eta) + \varepsilon^2\theta_2(\eta) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (21)

Substituting this into equation (7.2.15) and equating like powers of εignoring quadratic and higher order terms in ε, we obtain

$(1+Nr)\theta_{0\eta\eta} + Pr\, f\, \theta_{0\eta} - Pr\big(rf_\eta - \beta\big)\theta_0 = 0$ (22)

with the boundary conditions

$\theta_0(0) = 1, \theta_0(\infty) = 0$(23)

and

$(1+Nr)\theta_{1\eta\eta} + Pr\, f\, \theta_{1\eta} - Pr\big(rf_\eta - \beta\big)\theta_1 = -\theta_0\theta_{0\eta\eta} - \theta_{0\eta}^2$(24)

with the boundary conditions

$\theta_1(0) = 0, \theta_1(\infty) = 0$(25)

Equation for $\boldsymbol{\theta_0}$ can be solved explicitly in terms of Kummer's function and is given by

$\boldsymbol{\theta_0(\eta) = C_0\, exp(-\alpha(a_0 + b_0)\eta/2)\, F(a_1, b_1; z)}$(26)

Where $a_0 = {Pr}/{\alpha^2(1+Nr)}$,$b_0 = a_0\sqrt{1-\frac{4\beta}{a_0}}$

$z = -\frac{Pr}{\alpha^2 \, exp(-\alpha\eta)}$, $\quad a_1 = (a_0 + b_0 - 2r)/2(1+Nr)$

$b_1 = 1 + b_0$, $\quad \frac{1}{C_0} = F\left(a_1, b_1; -\frac{Pr}{\alpha^2}()\right)$

A similar analysis may be carried out for equation (19) (PHF case). We obtain

$g_0(\eta) = C_1 \, exp(-\alpha(a_0 + b_0)\eta/2) \, F(a_1, b_1; z)$ (27)

Where $g = g_0(\eta) + \varepsilon g_1(\eta) + \varepsilon^2 g_2(\eta) +$.............................and a_0,b_0,a_1,b_1 z are as above and

$\frac{1}{C_1} = \alpha\left(\frac{a_0+b_0}{2}\right) F\left((a_1, b_1; -\frac{Pr}{\alpha^2}) - \left((\frac{a_1 Pr}{b_1\alpha}) F\left(a_1 + 1, b_1 + 1; -\frac{Pr}{\alpha^2}\right)\right)\right)$(28)

Numerical Solution

We now analyze the equation (24) (PST Case), which gives the first order correction term $\varepsilon\vartheta_1$. Note that (24) is linear and inhomogeneous and therefore it is possible to obtain a power series solution for ϑ_1. However, it becomes very tedious to obtain various values of ϑ_1 using this power series solution. Instead, we employ the shooting technique with fourth order Runge-Kutta integration scheme to find θ_1. Similar procedure is applied to the PHF case also.

Results and Discussion

In order to understand the physical situation of the problem, we have computed the numerical values of the temperature, for different values of physical parameters. The obtained numerical values are illustrated in Figs 1-6. Numerical values of wall temperature gradient in PST case and wall temperature flux in PHF case are illustrated in Table 1. In the absence of thermal radiation and constant thermal conductivity.Figure 1 It is observed that the temperature profile decreases in the boundary layer with the increase of distance from the boundary. It is also noticed that the temperature distribution is unchanged at the wall with the change of physical parameters. However, it tends to zero in the free stream. The effect of visco-elastic parameter is to increase the temperature profile, and this is even true in the presence of thermal conductivity. This is since the thickening of thermal boundary layer occurs due to the increase of visco-elastic normal stress.

Figure 2 It is noticed that the effect of magnetic parameter is to increase the temperature profile $\theta(\eta)$ in the boundary layer. This is because the introduction of transverse magnetic field to an electrically conducting fluid gives rise to a resistive type of force known as Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer and to increase its temperature profile. Also, the effect on the flow and thermal fields become more so as the strength of the magnetic field increases.

Figure 3 we observe that the increase in wall temperature parameter (r) leads the temperature profile $\theta(\eta)$ to decrease and the magnitude of wall temperature gradient increases with wall temperature. This is since, when r > 0, heat flows from the stretching sheet into the ambient medium and, when r < 0, the temperature gradient is positive and heat flows into the stretching sheet from the ambient medium.

Figure 4 shows the effect of thermal radiation on temperature profile $\theta(\eta)$ in the boundary layer. It is observed that the increase in thermal radiation parameter (Nr) produces a significant increase in the thickness of the thermal boundary layer of the fluid and so the temperature profiles $\theta(\eta)$ increases.

Figure 5 demonstrates the effect of Prandtl number on temperature profile in the boundary layer. It is seen that the effect of Prandtl number is to decrease the temperature profile in the boundary layer. This is because thermal boundary layer thickness decreases with increase in Prandtl number.

Figure 6 The effect of heat source / sink parameter (β) on temperature profile $\theta(\eta)$ in the boundary layer is shown in Fig.6. It is observed that the effect of heat source (β> 0) in the boundary layer generates the energy, which causes the temperature to increase, while the presence of heat sink (β< 0) in the boundary layer absorbs the energy, which causes the temperature to decrease.

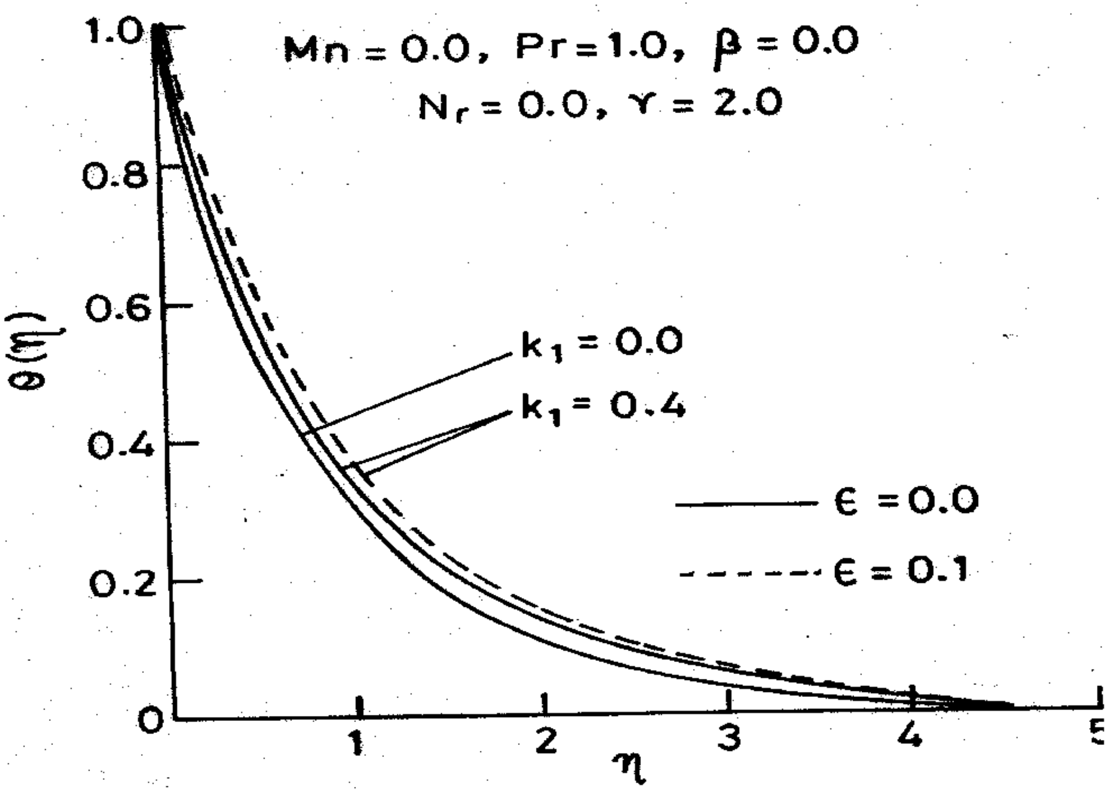


Fig .1. Variation of θ(η) vs .η for diffierent values of visco-elastic parameter.

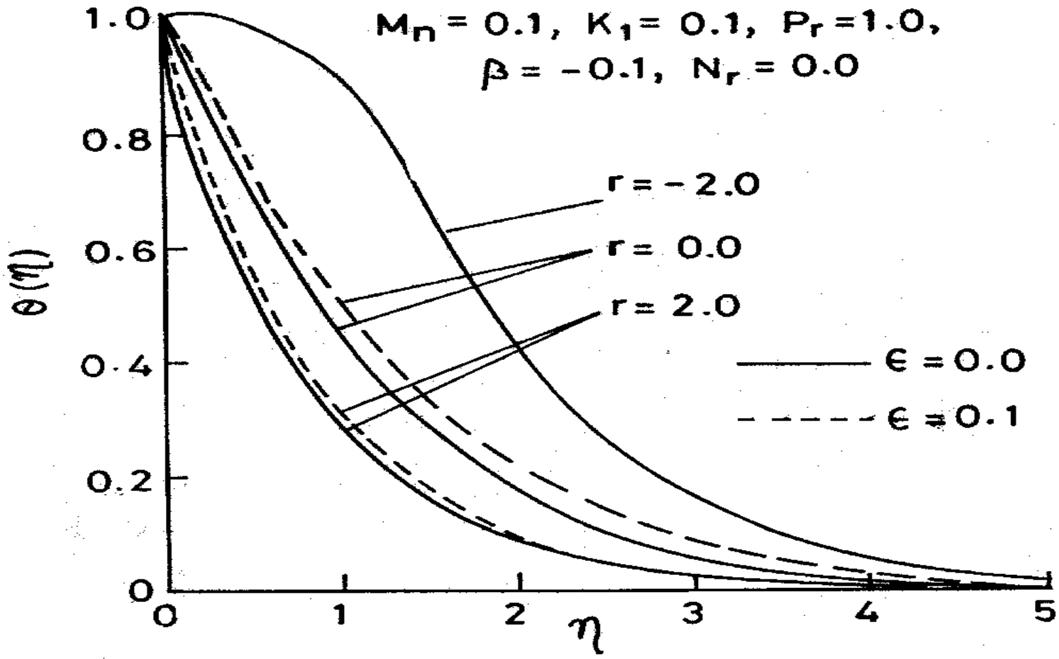


Fig. 3. Variation of $\theta(\eta)$ vs η for diffierent values of wall temperature parameter.

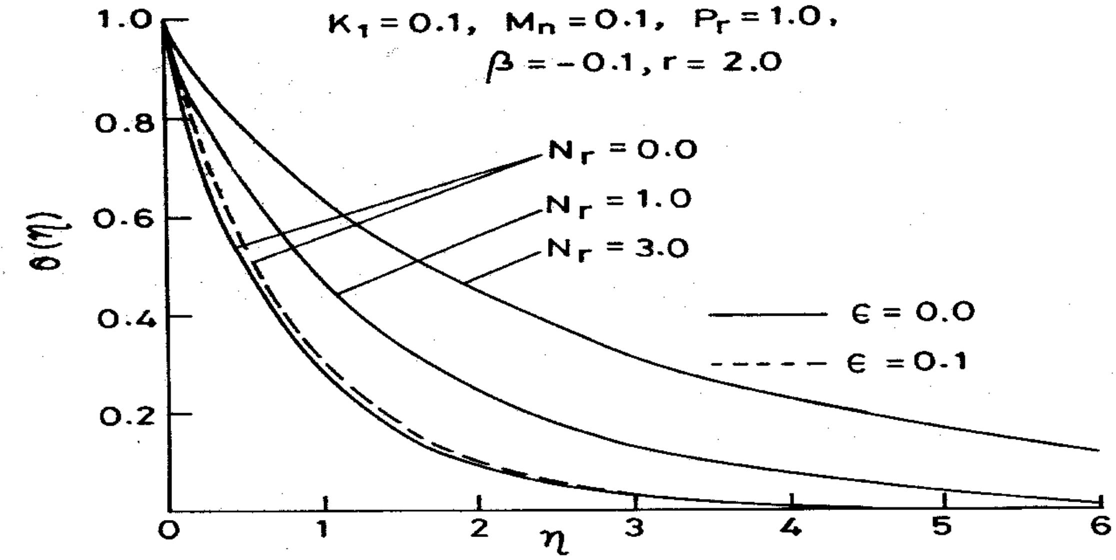


Fig. 4. Variation of $\theta(\eta)$ vs η for diffierent values of thermal radiation parameter.

Variation of θ(η) vs. η for diffierent values of heat source/sink parameter.

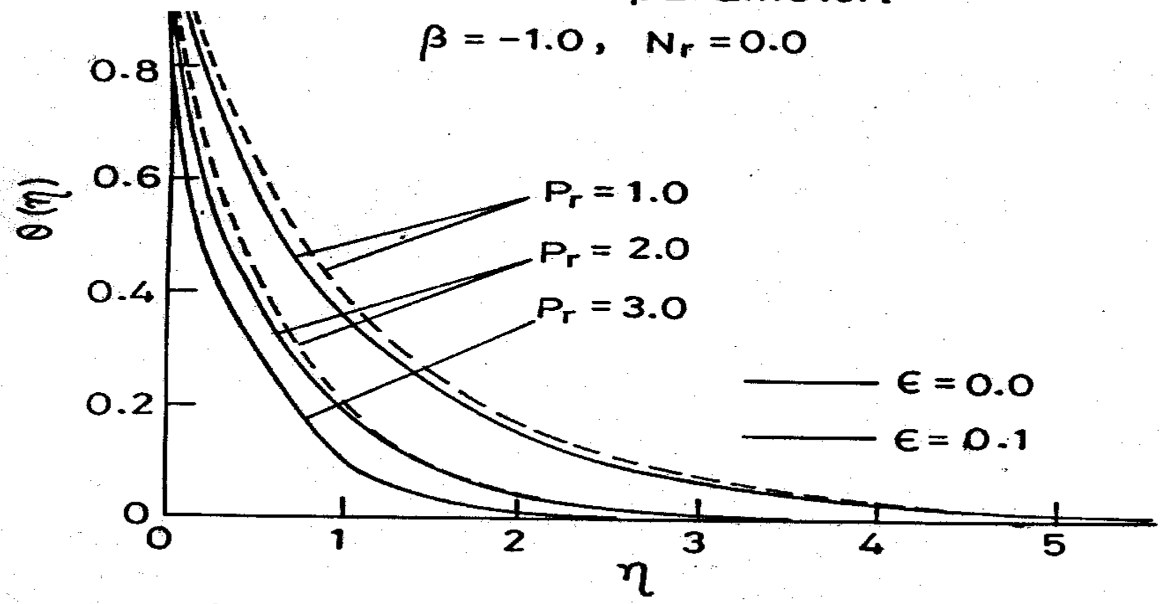
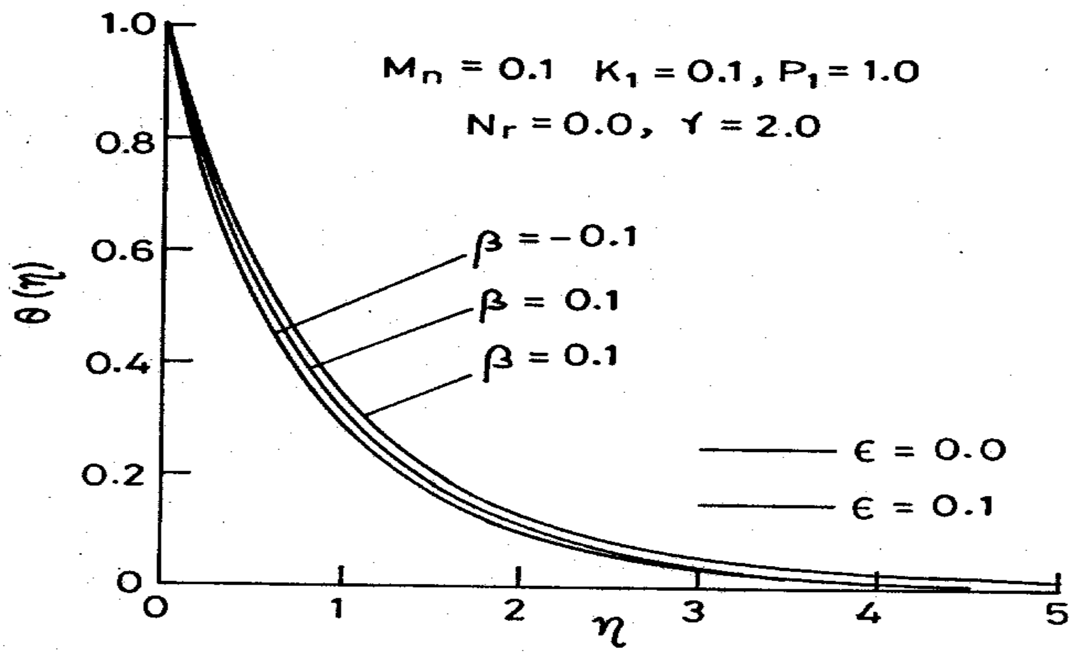


Fig. 5 Variation of θ(η) vs η for diffierent val values of prandtl number.



lue Fig. 6. Variation of θ(η) vs. η for diffierent values of heat source/sink parameter.

Pr	k1	Mn	β	Nr	r	θ_s'(0) in PST case			
						ε = 0	ε = 0.1	ε = 0.2	ε = 0.3
1.0	0.0	0.0	0.0	0.0	2.0	-1.3333	-1.2616	-1.1899	-1.1182
	0.2					-1.3000	-1.2339	-1.1678	-1.1017
	0.4					-1.2508	-1.1910	-1.1313	-1.0715
1.0	0.0	1.0	0.0	0.0	2.0	-1.2187	-1.1597	-1.1036	-1.0476
	0.2					-1.1678	-1.1170	-1.0653	-1.0136
	0.4					-1.1014	-1.0555	-1.0089	-0.9626
1.0	0.0	2.0	-0.1	0.0	2.0	-1.1993	-1.1403	-1.0812	-1.0221
	0.2					-1.1539	-1.0979	-1.0419	-0.9860
	0.4					-1.0924	-1.0401	-0.9879	-0.9357
1.0	0.1	0.1	-0.1	0.0	2.0	-1.3522	-1.2779	-1.2037	-1.1295
			0.0			-1.3035	-1.2369	-1.1703	-1.1036
			0.1			-1.2495	-1.1937	-1.1379	-1.0821
1.0	0.1	0.1	-0.1	0.0	-2.0	0.5979	1.4335	2.2871	3.1408
					0.0	-0.6524	-0.5552	-0.4581	-0.3609
					2.0	-1.3522	-1.2779	-1.2037	-1.1295
1.0	0.1	0.1	0.1	-0.1	0.0	-1.3522	-1.2779	-1.2037	-1.1295
				1.0		-0.7471	-0.7328	-0.7185	-0.7042
				3,0		-0.4108	-0.4108	-0.4096	-0.4070

Table 1: Wall Temperature Gradients θ_s'(0) for the case of Prescribed Surface Temperature

Pr	k1	Mn	β	Nr	r	g(0) in PST case			
						ε = 0	ε = 0.1	ε = 0.2	ε = 0.3
1.0	0.0	0.0	-0.1	0.0	2.0	0.7215	0.7758	0.8264	0.8771
	0.2					0.7412	0.7946	0.8479	0.9013
	0.4					0.7660	0.8326	0.8811	0.9387
1.0	0.0	1.0	-0.1	0.0	2.0	0.7843	0.8451	0.9059	0.9667
	0.2					0.8099	0.8753	0.9407	1.0061
	0.4					0.8487	0.9212	0.9938	1.066
1.0	0.0	2.0	-0.1	0.0	2.0	1.1948	1.2592	1.3235	1.3878
	0.2					1.2760	1.3475	1.4191	1.4907
	0.4					1.4428	1.5270	1.6113	1.6956
1.0	0.1	0.1	-0.1	0.0	2.0	0.7395	0.7925	0.8456	0.8987
			0.0	1.0		1.1948	1.2592	1.3235	1.3878
			0.1	3.0		1.9933	2.0698	2.1464	2.2229
1.0	0.1	0.1	-0.1	0.0	-1.0	6.5245	7.6535	8.7824	9.9114
					0.0	1.5327	1.7190	1.9052	2.0915
					2.0	0.7395	0.7925	0.8455	0.8987
1.0	0.2	1.0	-0.1	0.0	2.0	1.3361	1.4184	1.5006	1.5829
2.0						0.8474	0.8813	0.9151	0.9490
3.0						0.6569	0.6772	0.6974	0.7177

Table 2: Wall Temperature Gradients g(0) for the case of Prescribed Surface Temperature

Conclusion

1.Fromthe Tables 1 and 2, we observe that the effect of visco-elastic parameter is to increase the wall temperature gradient in PST case and the wall temperature in PHF case

2.The wall gradients of PST and PHF cases increases as the thermal radiation parameter increases that can be observed in Table 1 and 2

3. The effect of heat source is more pronounced as compared to that of heat sink

4.Table 1 and 2 shows the heat transfer increases with Prandtl number because a higher Prandtl number fluid has relatively lower thermal conductivity. Thusfor PST case this results in the reduction of the thermal boundary layer thickness and increase in the increase in the heat transfer at the wall. For PHF case, the temperature at the wall reduces as the Prandtl number increases because of the cooling effect on the surface caused by the increase in Prandtl number.

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